1. Exercise 2.5.2 on page 79 of Hopcroft et al.

Consider the following ε-NFA.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow p$</td>
<td>${q, r}$</td>
<td>$\emptyset$</td>
<td>${q}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\emptyset$</td>
<td>${p}$</td>
<td>${r}$</td>
</tr>
<tr>
<td>$\ast r$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

(a) Compute the ε-closure of each state.

- $p \rightarrow \{p, q, r\}$
- $q \rightarrow \{q\}$
- $r \rightarrow \{r\}$

(b) Give all the strings of length three or less accepted by the automaton.

- $\epsilon, a, b, c, ba, bb, bc, ca, cb, cc, aaa, aab, aac, aba, abb, baa, bab, bac, bca, bcb, bcc, caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc$

(c) Convert the automaton to a DFA. (Please construct the table, and then draw the diagram.)

We first construct the table below:

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow \ast {p, q, r}$</td>
<td>${p, q, r}$</td>
<td>${q, r}$</td>
<td>${p, q, r}$</td>
</tr>
<tr>
<td>$\ast {q, r}$</td>
<td>${p, q, r}$</td>
<td>${r}$</td>
<td>${p, q, r}$</td>
</tr>
<tr>
<td>$\ast {r}$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Diagram of the DFA: [diagram image]
2. Exercise 3.1.1 on page 91 of Hopcroft et al.

Write regular expressions for the following languages.

(a) The set of strings over alphabet \{a, b, c\} containing at least one a and at least one b.

\[(A + B + C)^*(A(A + B + C)^*B + B(A + B + C)^*A)(A + B + C)^*\]

(b) The set of strings of 0’s and 1’s whose tenth symbol from the right end is 1.

\[(0 + 1)^*1(0 + 1)^9\]

(c) The set of strings of 0’s and 1’s with at most one pair of consecutive 1’s.

\[(0 + 10)^*(11 + \epsilon)(0 + 10)^*\]

3. Exercise 3.1.4 on page 92 of Hopcroft et al.

Give English descriptions of the languages of the following regular expressions.

(a) \((1 + \epsilon)(00^*1)*0^*\)

This is the language of strings with no two consecutive 1’s.

(b) \((0^*1^*)^*000(0 + 1)^*\)

This is the language of strings with three consecutive 0’s.

(c) \((0 + 10)^*1^*\)

This is the language of strings in which there are no two consecutive 1’s, except for possibly a string of 1’s at the end.

4. Exercise 3.2.1 on page 107 of Hopcroft et al.

Here is a transition table for a DFA:

<table>
<thead>
<tr>
<th>→</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rightarrow q_1)</td>
<td>(q_2)</td>
<td>(q_1)</td>
</tr>
<tr>
<td>(q_2)</td>
<td>(q_3)</td>
<td>(q_1)</td>
</tr>
<tr>
<td>(q_3)</td>
<td>(q_3)</td>
<td>(q_2)</td>
</tr>
</tbody>
</table>

(a) Give all the regular expressions \(R_{ij}^{(0)}\). Note: Think of state \(q_i\) as if it were the state with integer number \(i\).
\[
R_{11}^{(0)} = 1 + \epsilon \\
R_{12}^{(0)} = 0 \\
R_{13}^{(0)} = \emptyset \\
R_{21}^{(0)} = 1 \\
R_{22}^{(0)} = \epsilon \\
R_{23}^{(0)} = 0 \\
R_{31}^{(0)} = \emptyset \\
R_{32}^{(0)} = 1 \\
R_{33}^{(0)} = 0 + \epsilon \\
\]

(b) Give all the regular expressions \( R_{ij}^{(1)} \). Try to simplify the expressions as much as possible.
\( R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} \\
= (1 + \epsilon) + (1 + \epsilon)(1 + \epsilon)^*(1 + \epsilon) \\
= 1^* \\
R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\
= 0 + (1 + \epsilon)(1 + \epsilon)^* 0 \\
= 1^*0 \\
R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{13}^{(0)} \\
= \emptyset \\
R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} \\
= 1 + 1(1 + \epsilon)^*(1 + \epsilon) \\
= 1^+ \\
R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\
= \epsilon + 1(1^*)0 \\
= \epsilon + 1^+0 \\
R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{13}^{(0)} \\
= 0 \\
R_{31}^{(1)} = R_{31}^{(0)} + R_{31}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} \\
= \emptyset \\
R_{32}^{(1)} = R_{32}^{(0)} + R_{31}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\
= 1 \\
R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)} (R_{11}^{(0)})^* R_{13}^{(0)} \\
= 0 + \epsilon \\
\)

(c) Give all the regular expressions \( R_{ij}^{(2)} \). Try to simplify as much as possible.
\[ R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)} \]
\[ = 1^* + 1^* 0(\epsilon + 1^+ 0)^* 1^+ \]
\[ = (1 + 01)^* \]
\[ R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \]
\[ = R_{12}^{(1)} (R_{22}^{(1)})^* \]
\[ = 1^* 0(\epsilon + 1^+ 0)^* \]
\[ = (1 + 01)^* 0 \]
\[ R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} \]
\[ = 0 + 1^* 0(\epsilon + 1^+ 0)^* 0 \]
\[ = (1 + 01)^* 00 \]
\[ R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)} \]
\[ = (R_{22}^{(1)})^* R_{21}^{(1)} \]
\[ = (\epsilon + 1^+ 0)1^+ \]
\[ = 1^+ (\epsilon + 01^+) \]
\[ R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \]
\[ = (R_{22}^{(1)})^+ \]
\[ = (\epsilon + 1^+ 0)^+ \]
\[ = (1^+ 0)^* \]
\[ R_{23}^{(2)} = R_{23}^{(1)} + R_{22}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} \]
\[ = (R_{22}^{(2)})^* R_{23}^{(1)} \]
\[ = (\epsilon + 1^+ 0)^* 0 \]
\[ = (1^+ 0)^* 0 \]
\[ R_{31}^{(2)} = R_{31}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{21}^{(1)} \]
\[ = 0 + 1(\epsilon + 1^+ 0)^* 1^+ \]
\[ = 1(1^+ 0)^* 1^+ \]
\[ R_{32}^{(2)} = R_{32}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \]
\[ = 1 + 1(\epsilon + 1^+ 0)^* \]
\[ = 1(1^+ 0)^* \]
\[ R_{33}^{(2)} = R_{33}^{(1)} + R_{32}^{(1)} (R_{22}^{(1)})^* R_{23}^{(1)} \]
\[ = (0 + \epsilon) + 1(\epsilon + 1^+ 0)^* 0 \]
\[ = 0 + 1(1^+ 0)^* 0 + \epsilon \]

(d) Give a regular expression for the language of the automaton.
The language of our DFA is $R_{13}^{(3)}$.

$$R_{13}^{(3)} = R_{13}^{(2)} + R_{13}^{(2)}(R_{33}^{(2)})^*R_{33}^{(2)}$$
$$= R_{13}^{(2)}(R_{33}^{(2)})^*$$
$$= (1 + 01)^*00(0 + 1(1^+0)^*0 + \epsilon)^*$$
$$= (1 + 01)^*00(0 + 1(1^+0)^*0)^*$$

(e) Construct the transition diagram for the DFA and give a regular expression for its language by eliminating state $q_2$.

The transition diagram is:

```
+---+----+---+
| 1 | 0  | 0 |
| q1 | q2 | q3 |
+---+----+---+
```

When we eliminate $q_2$, we get the following diagram:

```
+---+----+---+
| 1+01 | 00 | 0+01 |
| q1   | q3 |
+---+----+---+
```

This gives us the following regular expression for the language of our DFA:

$$[1 + 01 + 00(0 + 10)^*11]^*00(0 + 10)^*$$

5. Exercise 3.2.4 on page 108 of Hopcroft et al.
Convert the following regular expressions to NFA’s with ε-transitions. (I’ve simplified my solutions somewhat, but some students may turn in equivalent solutions that are more complicated because they followed the book exactly, which is fine.)

(a) \(01^*\).

![Diagram for (a)](image1)

(b) \((0 + 1)01\).

![Diagram for (b)](image2)

(c) \(00(0 + 1)^*\).

![Diagram for (c)](image3)