Finite Automata

Motivation

Examples

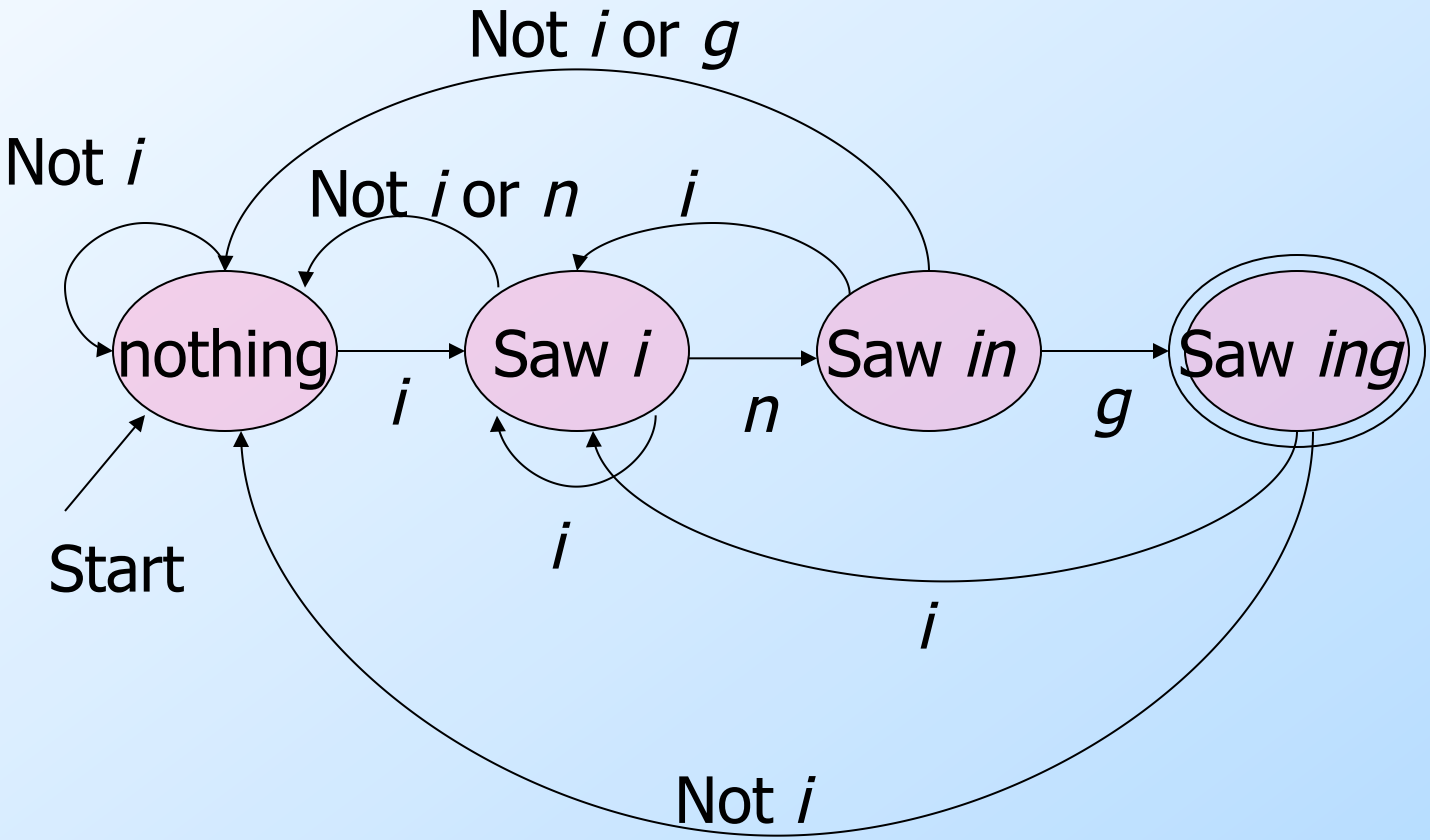
Informal Explanation

- Finite automata are finite collections of states with transition rules that take you from one state to another.
- Original application was sequential switching circuits, where the “state” was the settings of internal bits.
- Today, several kinds of software can be modeled by Finite Automata.

Representing Finite Automata

- Simplest representation is often a graph.
  - Nodes = states.
  - Arcs indicate state transitions.
  - Labels on arcs tell what causes the transition.

Example: Recognizing Strings Ending in "ing"



Automata to Code (by hand)

- In C/C++/Java:
  1. Initialize state q to start state.
  2. Loop through the string one character at a time.
  3. Make a switch statement with a case for each state for q, where each case sets q according to the transitions for that state.
  4. Accept if you end in a final state.

Example in Java

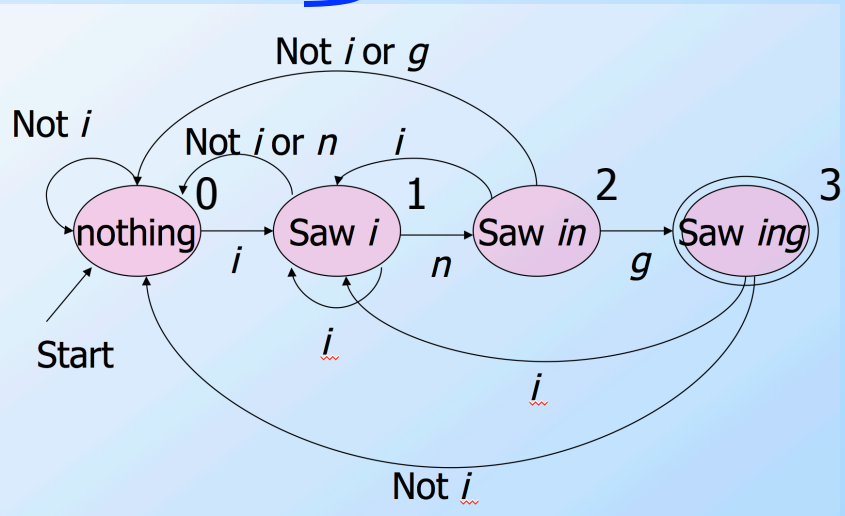
```
Scanner scan = new Scanner(System.in);
String s = scan.next();
int q = 0;
for (char c : s.toCharArray()) {
  switch (q) {
    case 0: q = (c=='i')? 1 : 0; break;
    case 1: q = (c=='n')? 2 : ((c=='i')? 1 : 0); break;
    case 2: q = (c=='g')? 3 : ((c=='i')? 1 : 0); break;
    case 3: q = (c=='i')? 1 : 0;
    }
}
if (q==3)
  System.out.println("accept.");
else
  System.out.println("reject.");
```

Start state

Loop through string s

Transitions

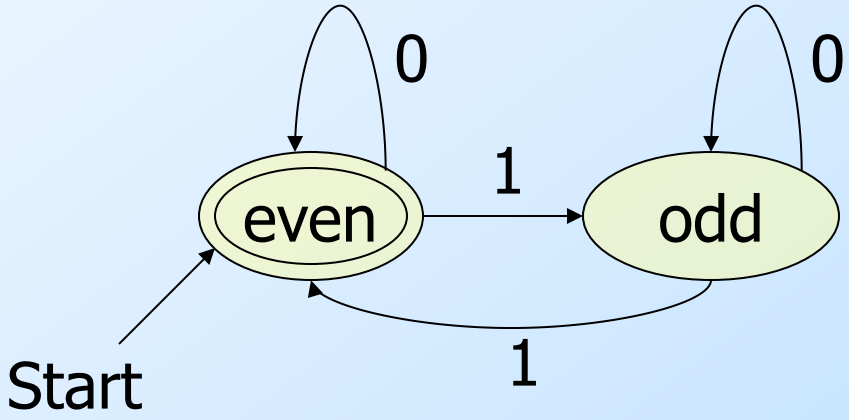
Final state



Automata to Code – General

- It would be nice to have an automatic way to generate such code…
- Rather than do it by hand, a code generator takes a "regular expression" describing the pattern(s) you are looking for and produces the code for it.
  - Example: `.*ing` works in grep.

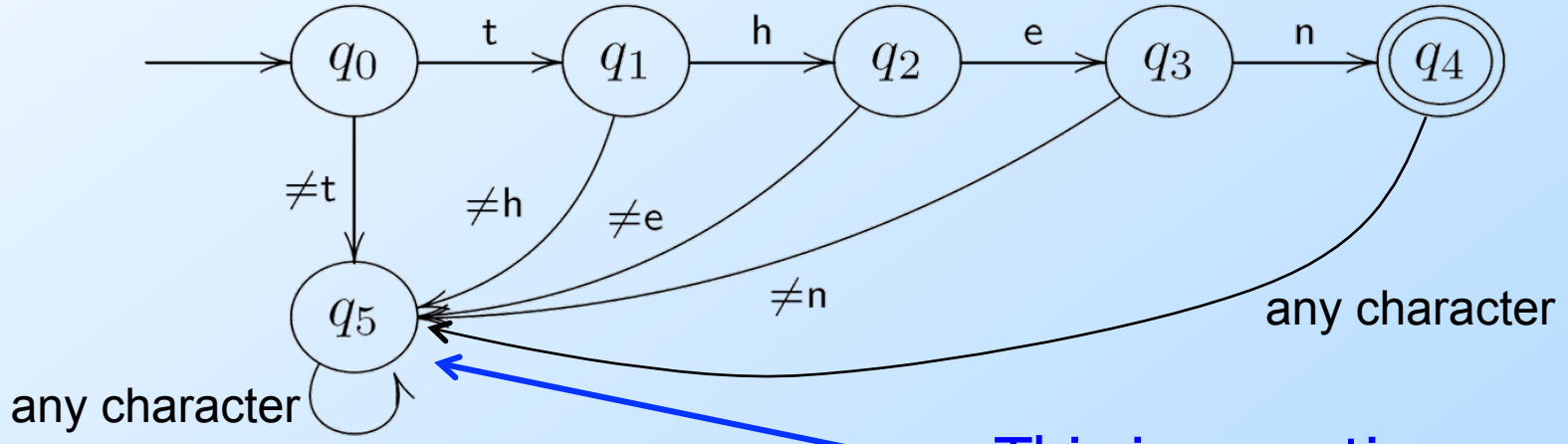
Example: An Even Number of 1's



- How would it look to accept a number of 1's that is a multiple of 3?

Password/Keyword Example

It reads the word and accepts it if it stops in an accepting state

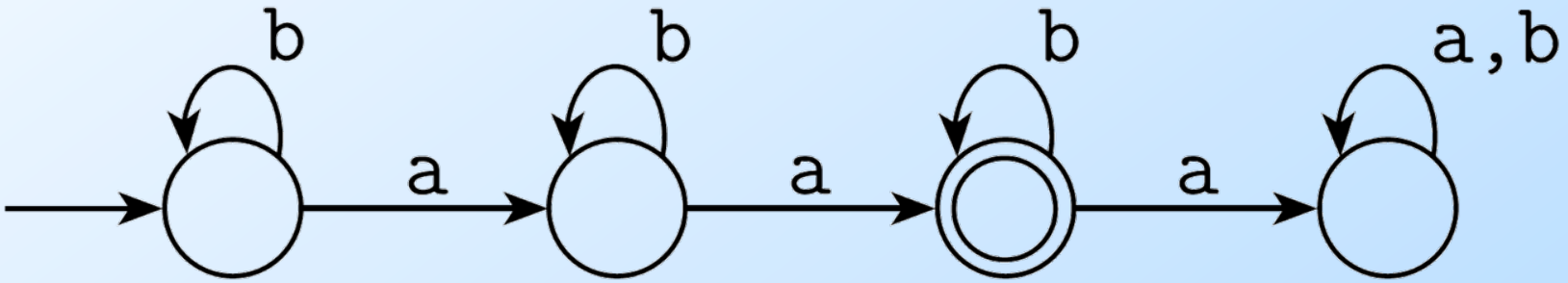


Only the word then is accepted

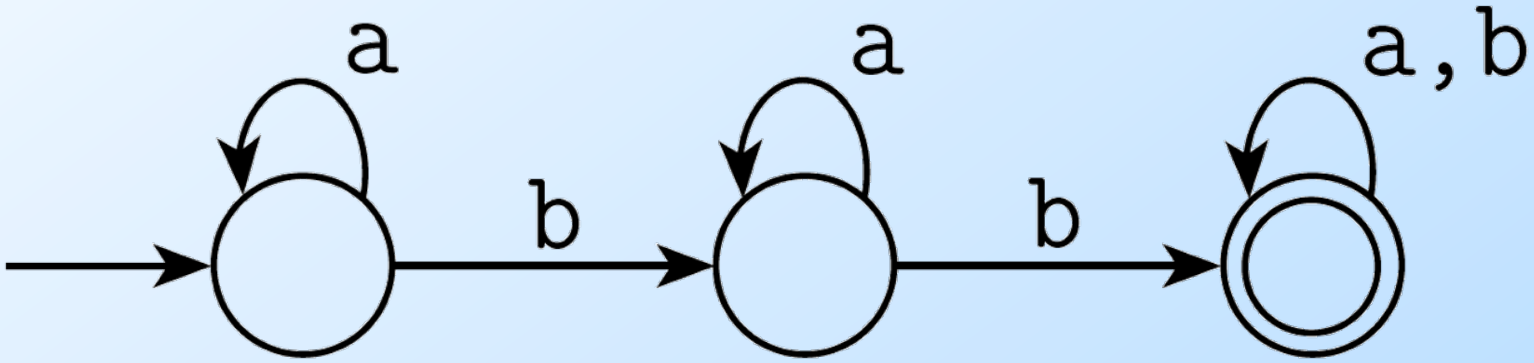
This is sometimes called a **dead** state.

BTW, there is a potential security risk on the password application if this finite automaton reports failure too quickly.

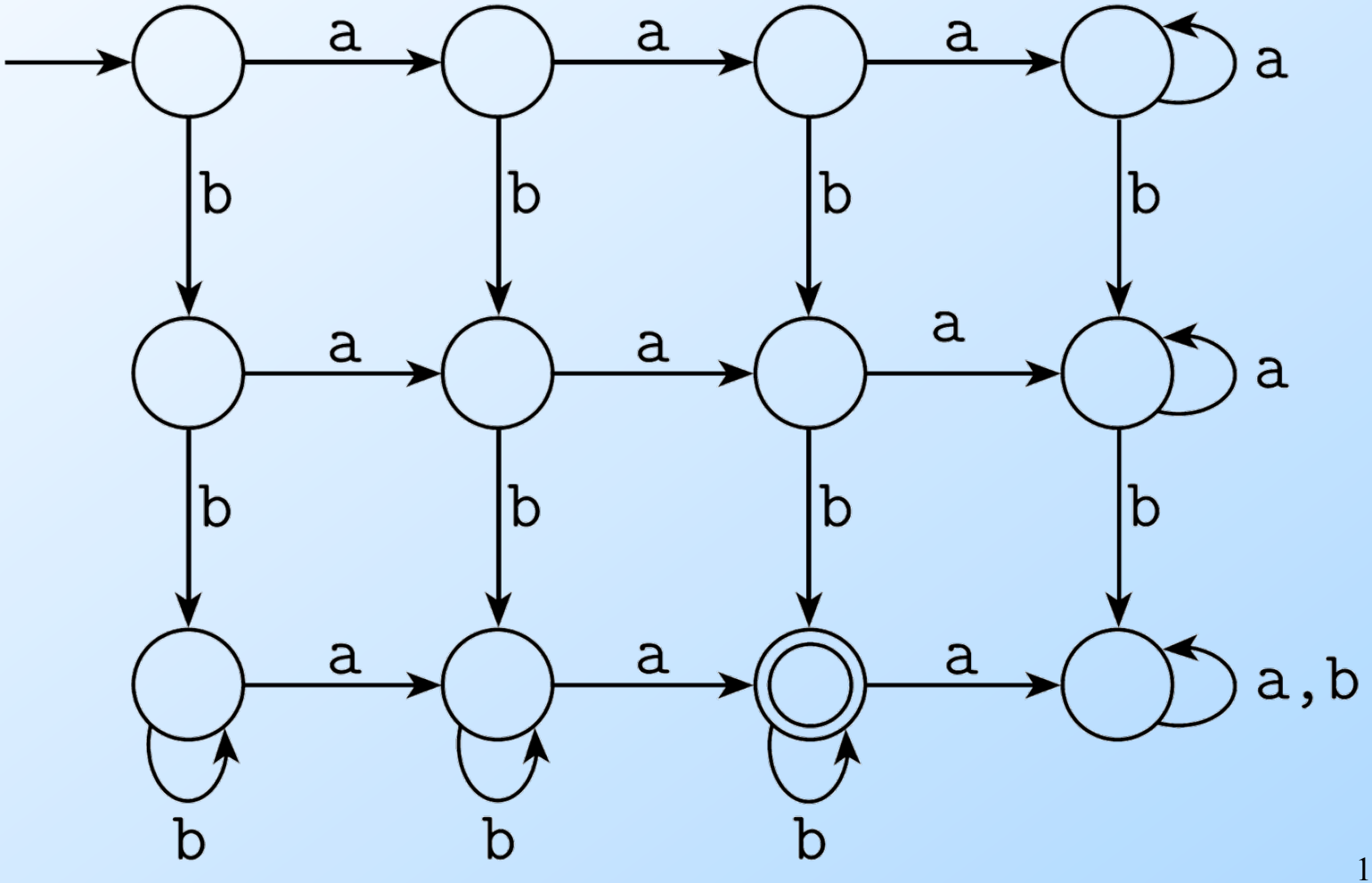
Exactly Two a's



At Least Two b's

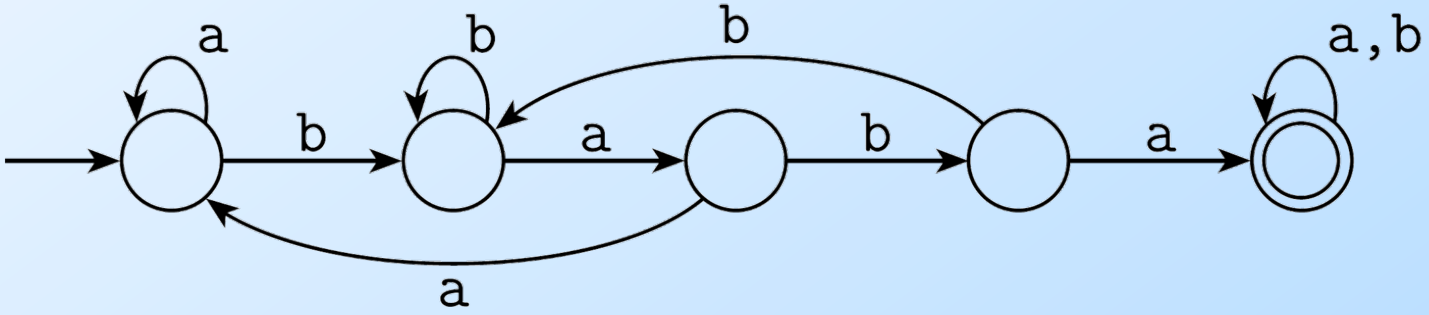


Exactly two a's and at least two b's

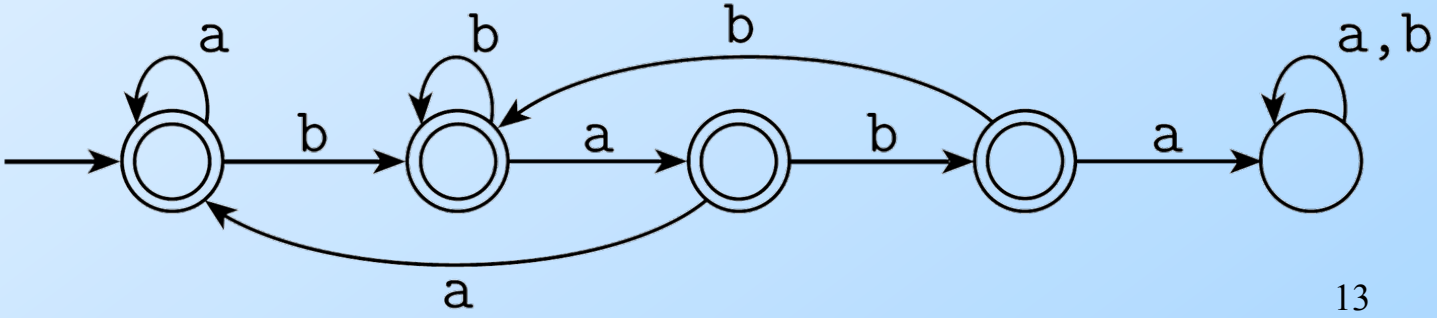


Containing Substrings or Not

- Contains baba:



- Does not contain baba:



General Comments

- Some things are easy with finite automata:
  - Substrings (…abcabc…)
  - Subsequences (…a…b…c…b…a…)
  - Modular counting (odd number of 1's)
- Some things are impossible with finite automata (we will prove this later):
  - An equal number of a's and b's
  - More 0's than 1's
- But when they **can** be used, they are fast.