1. **30 points.** Define each of the following terms (using at most 2 sentences each):

   (a) convex hull,
   
   (b) planar subdivision,
   
   (c) trapezoidal decomposition.

2. **30 points.** Describe an efficient method for finding the convex hull of \( n \) points in the plane.

   **NOTE:** For the remainder of this exam you may assume that you have a subroutine for any problem we discussed in class, provided you can correctly characterize its performance bounds.

3. **30 points.** Describe an efficient algorithm for determining the width of a set \( S \) of \( n \) points in the plane. Recall that the width of a point set is the smallest distance between two parallel lines that contain the points of \( S \) between them.

4. **30 points.** Suppose you are given a set \( S \) of \( n \) line segments in the plane, such that each makes a positive angle with the \( x \)-axis of either 30\(^\circ\) or 60\(^\circ\) (so there are only two possible slopes for the lines in \( S \)). Sketch an efficient algorithm for finding all the pairs of intersecting segments in \( S \). What is the running time of your method?

5. **30 points.** Describe a dynamic data structure that can store a set of \( n \) intervals in \( \mathbb{R} \) that all have integer endpoints in the range \([1, N]\). Mention how your structure efficiently supports each of the following operations (hint: think of left and right endpoints separately):

   (a) \( \text{Insert}([a, b]) \): insert a new interval \([a, b]\) to the set.
   
   (b) \( \text{Delete}([a, b]) \): remove an interval \([a, b]\) from the set.
   
   (c) \( \text{Outside}([a, b]) \): report all the intervals in the set that do not intersect \([a, b]\).

   What is the running time for each method?