Please answer the following questions, each of which is worth 10 points.

1. (CS 163 students only) Using the graph at the following website, number each vertex according to a topological ordering.

   http://www.cs.unc.edu/~stotts/145/homes/qualsched/pert_chart.jpg

2. (CS 163 students only) Using the graph at the following website, darken in each edge of the shortest path tree from the node, A.

   https://i.stack.imgur.com/kJF5K.png

   For each other vertex, \( v \), in this graph, show the values of the label, \( D[v] \), as it would be set during Dijkstra’s algorithm (starting with \( +\infty \)), and going through each update to each such label until it has its final value.

3. Suppose you are given a connected weighted undirected graph, \( G \), with \( n \) vertices and \( m \) edges, such that the weight of each edge in \( G \) is an integer in the interval \([1, c]\), for a fixed constant \( c > 0 \). Show how to solve the single-source shortest-paths problem, for any given vertex \( v \), in \( G \), in time \( O(n + m) \).

4. In a side-scrolling video game, a character moves through an environment from, say, left-to-right, while encountering obstacles, attackers, and prizes. The goal is to avoid or destroy the obstacles, defeat or avoid the attackers, and collect as many prizes as possible while moving from a starting position to an ending position. We can model such a game with a graph, \( G \), where each vertex is a game position, given as an \((x, y)\) point in the plane, and two such vertices, \( v \) and \( w \), are connected by an edge, given as a straight line segment, if there is a single movement that connects \( v \) and \( w \). Furthermore, we can define the cost, \( c(e) \), of an edge to be a combination of the time, health points, prizes, etc., that it costs our character to move along the edge \( e \) (where earning a prize on this edge would be modeled as a negative term in this cost). A path, \( P \), in \( G \) is monotone if traversing \( P \) involves a continuous sequence of left-to-right movements, with no right-to-left moves. Thus, we can model an optimal solution to such a side-scrolling computer game in terms of finding a minimum-cost monotone path in the graph, \( G \), that represents this game. Describe and analyze an efficient algorithm for finding a minimum-cost monotone path in such a graph, \( G \).

5. Suppose that CONTROL, a secret U.S. government counterintelligence agency based in Washington, D.C., has build a communication network that links \( n \) stations spread across the world using \( m \) communication channels between pairs
of stations (i.e., edges). Suppose further that the evil spy agency, KAOS, is able to eavesdrop on some number, $k$, of these channels and that CONTROL knows the $k$ channels that have been compromised. Now, CONTROL has a message, $M$, that it wants to send from its headquarters station, $s$, to one of its field stations, $t$. The problem is that the message is super secret and should traverse a path that minimizes the number of compromised edges that occur along this path. Explain how to model this problem as a shortest-path problem, and describe and analyze an efficient algorithm to solve it.

6. (CS 265 students only) Show that it is possible to count the total number of paths from a source vertex, $s$, to a sink vertex, $t$, in a directed acyclic graph, $G$, with $n$ vertices and $m$ edges using $O(n + m)$ additions. Also, show that there is a graph, $G$, where this number is at least $2^{n/2}$.

7. (CS 265 students only) Design an efficient algorithm for finding a longest directed path from a vertex $s$ to a vertex $t$ of a directed acyclic weighted graph, $G$. Specify the graph representation used and any auxiliary data structures used. Also, analyze the time complexity of your algorithm.