Please answer the following questions, each of which is worth 10 points.

1. (CS 163 students only) Show that the Grötzsch graph (see link below) can be (vertex) colored using 4 colors.

   https://en.wikipedia.org/wiki/Gr%C3%B6tzsch_graph

2. (CS 265 students only) Show that the Grötzsch graph (see link above) cannot be (vertex) colored using 3 colors.

3. Prove that the chromatic number of a disconnected graph is the largest chromatic number of any of its connected components.

4. What is the chromatic number of a graph obtained from $K_n$ by removing one edge, where $n \geq 3$?

   https://en.wikipedia.org/wiki/Complete_graph

5. The following committees need to have meetings scheduled:

   A={Smith, Jones, Brown, Green}
   B={Jones, Wagner, Chase}
   C={Harris, Oliver}
   D={Harris, Jones, Mason}
   E={Oliver, Cummings, Larson}

   Are three meeting times (using multiple rooms) sufficient to schedule the committees so that no member has to be at two meetings simultaneously? Justify your answer. (Hint: explain how to model this problem using a graph.)

6. The following tours of garbage trucks in Orange County are being considered by the Orange County waste management company.

   Tour 1: The Spectrum, Diamond Jamboree, and the Great Park
   Tour 2: The Bluffs, the Great Park, the Spectrum
   Tour 3: Segerstrom Center, Hoag Hospital, and UCI
   Tour 4: University Center and the Great Park
   Tour 5: University Center, Disneyland, and the Spectrum
   Tour 6: Segerstrom Center, Angels Stadium, and Hoag Hospital
   Tour 7: Disneyland, Crystal Cove Beach, and Hoag Hospital

   Assuming the sanitation workers refuse to work more than three days a week, can these tours be partitioned so that no site is visited more than once on a given day? Justify your answer. (Hint: explain how to model this problem using a graph.)