

Homework 1 ICS 163 Graph Algorithms

Due: Wednesday, January 15, 2003, in class

Please answer the following questions, each of which is worth 10 points.

1. Draw a simple undirected graph G that has 12 vertices, 18 edges, and 3 connected components. Why would it be impossible to draw G with 3 connected components if G had 66 edges?
2. Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169. The course prerequisites are:
 - LA15: (none)
 - LA16: LA15
 - LA22: (none)
 - LA31: LA15
 - LA32: LA16, LA31
 - LA126: LA22, LA32
 - LA127: LA16
 - LA141: LA22, LA16
 - LA169: LA32.

Find a sequence of courses that allows Bob to satisfy all the prerequisites.

3. An *Euler tour* of an undirected graph G with n vertices and m edges is a cycle that traverses each edge of G exactly once. (That is, you can draw the edges of G without ever lifting your pencil.) Show that such a tour exists if and only if G is connected and every vertex has even degree.
4. Describe an $O(n + m)$ -time algorithm for finding an Euler tour of a connected undirected graph G such that every vertex has even degree.
5. Let G be an undirected graph with n vertices and m edges. Describe an algorithm running in $O(n + m)$ time that traverses each edge of G exactly once in each direction.