

# CS 264: Quantum Computation

## Homework 3

Fall 2009

Due: November 19, 2009

1. Let  $a|q$  and  $b|q$ . What is the Fourier transform mod  $q$  of the uniform superposition of all  $0 \leq x < q$  such that  $a|x$  or  $b|x$ ?
2. Prove that in the phase estimation procedure, if the eigenvalue is  $\lambda = e^{2\pi i\theta}$  then with high probability, the phase estimation procedure outputs  $a$  or  $a - 1$  such that  $\frac{a-1}{2^m} \leq \theta \leq \frac{a}{2^m}$ .
3. Let  $U$  be a unitary transformation implemented by a quantum circuit and let  $|\phi_i\rangle$  be its eigenvectors with eigenvalues  $\lambda_j = e^{2\pi i\theta_j}$ . Define  $\sqrt{U}$  to be the unitary transformation with the same eigenvectors but whose eigenvalues are  $\sqrt{\lambda_j}$ . Show that you can use phase estimation to implement an approximation to  $\sqrt{U}$ , which has eigenvectors  $|\phi_j\rangle$ , with eigenvalues  $\mu_j = e^{2\pi i\beta_j}$ , where  $|\theta_j/2 - \beta_j| \leq 1/2^q$ , where  $q$  is the number of qubits in the auxiliary register.