CS 264: Quantum Computation
Homework 4
Fall 2009

Due: December 11, 2009

1. Consider a function $f : \{0,1\}^n \rightarrow \{0,1\}^m$. Devise an quantum algorithm that will find $\min_y f(y)$. Your algorithm should compute the correct answer with probability at least 1/2. Express the complexity of your algorithm in terms of the number of gates in the circuit and the number of queries to f (although you need only express these in terms of the asymptotic dependency on m and n).

   You can assume the existence of a quantum algorithm for approximate counting. This algorithm will take a boolean function $g : \{0,1\}^n \rightarrow \{0,1\}$ given as a black box and will estimate the value of $M = |\{x \mid g(x) = 1\}|$. If $t = l + \lceil \log(2 + 1/(2\epsilon)) \rceil$, where l is a parameter denoting the accuracy in the estimate, the algorithm will use $O(t+n+1)$ qubits. The query complexity is $\Theta(t)$ and the computational complexity is $\Theta(n+t)$. The algorithm will produce $\tilde{M}$ such that with probability at least $1 - \epsilon$,

   $$|M - \tilde{M}| < \left(\sqrt{2MN} + \frac{N}{2^{l+1}} \right) 2^{-l}.$$

2. In class we discussed a method to simulate the dynamics of a quantum system when the Hamiltonian is the sum of local terms. We can expand the set of quantum systems that can be simulated to certain classes of non-local Hamiltonians. Give an algorithm to efficiently simulate the Hamiltonian

   $$H = \bigotimes_{j=1}^{n} \sigma_{s_j}^{j},$$

   where $s_j \in \{0,1,2,3\}$ and $\sigma_0, \sigma_1, \sigma_2, \sigma_3$ represent Pauli operators I, X, Y, Z. (Hint: start with $H = Z_1 \otimes Z_2 \otimes \cdots \otimes Z_n$.)

3. Recall, that we defined a continuous-time quantum walk on a graph G to be $|\psi(t)\rangle = e^{-iLt}|\psi(0)\rangle$, where L is the Laplacian of the graph.

   (a) Prove that for the quantum random walk on the hypercube, the distribution is uniform at $t = (\pi/4)k$ for any odd k if the state is measured in the vertex basis.

   (b) Since a quantum walk is a unitary process, we do not expect our quantum state to reach a limiting state. We can define the idea of a limiting distribution by picking a time t uniformly in the interval $[0, T]$, runing the walk starting at vertex a for time t and then measuring in the vertex basis. Define $p_{a \rightarrow b}(T)$ to be the probability of measuring vertex b. Prove that the quantum continuous time random walk on the hypercube does not converge. That is, if the walk starts in a single state $|a\rangle$, there exists some constant ϵ such that for any T_0, there exists some $T \geq T_0$ where

$$\sum_b |p_{a \to b}(T) - \frac{1}{2^n}| \geq \epsilon.$$

You can assume that the size of the hypercube on which the quantum walk is being performed is fixed.