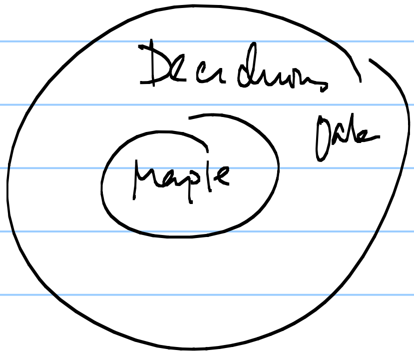
Note Title 10/2/2013

$\neg p \quad \bar{p} \quad p'$

$p \to q$

p: the tree is a maple
q: the tree is deciduous.

If the tree is a maple, it is deciduous.
The tree being a maple implies that it is deciduous.



The tree being a maple is sufficient for it to be deciduous.

The tree being deciduous is necessary for it to be a maple.

The tree is a maple only if it is deciduous.

$\neg \quad \wedge \quad \vee \quad \underbrace{\to \quad \leftrightarrow}$

$p \to (q \vee r) \qquad (\neg p \vee r) \to q$

$p = T, \quad q = F, \quad r = T$

$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

F T F T

F F

T

$(p \to q) \vee (\neg p \to r)$

$\underbrace{p \to q}_{F}$ T

T

You can access the internet only if you are a CS major or not freshman.

i ~~I~~: access int

c ~~C~~: CS major

f ~~F~~: freshman.

$i \to (c \vee \neg f)$

Anyone can ride the roller coaster as long as they are over four feet or over 16 yrs old.

r: allowed to ride

f: over 4 ft

s: over 16 yrs.

$r \leftrightarrow (f \vee s)$.

A says B is a knight.

B says the two (A&B) are different.

t ~~T~~ - knight

f ~~F~~ - knave.

A	B	A tells truth	B tell truth
t	t	t	f
t	f	f	t
f	t	t	t
f	f	f	f

Both knaves

$(p \leftrightarrow q) \vee (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \vee (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

tautology.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

$p \vee \neg p$: tautology

$p \wedge \neg p$: contradiction.

p	q	$p \to q$	$\neg q$	$p \wedge (p \to q) \wedge \neg q$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

contradiction

Two logical expressions are **logically equivalent** if they evaluate to the same truth value for every setting of logical variables

Claim: $\neg(p \vee q) \equiv \neg p \wedge \neg q$ De Morgan's Law.

p	$\neg p$	q	$\neg q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	F	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	T	F	T	T	T

Idempotent $\quad p \vee p \equiv p \qquad p \wedge p \equiv p.$

Associative $\quad (p \vee q) \vee r = p \vee (q \vee r) \qquad (p \wedge q) \wedge r = p \wedge (q \wedge r)$

Commutative $\quad p \wedge q \equiv q \wedge p \qquad p \vee q \equiv q \vee p.$

Distributive: $\quad p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$

Identity $\quad p \vee F \equiv p \qquad p \wedge F \equiv F$

$p \vee T \equiv T \qquad p \wedge T \equiv p.$

Involution $\quad \neg\neg p \equiv p.$

Complement $\quad p \vee \neg p \equiv T \qquad p \wedge \neg p \equiv F$

$\neg T \equiv F \qquad \neg F \equiv T$

De Morgan $\quad \neg(p \vee q) \equiv \neg p \wedge \neg q \qquad \neg(p \wedge q) \equiv \neg p \vee \neg q$

Conditional: $\quad p \to q \equiv \neg p \vee q \qquad p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$(\underbrace{\neg p \wedge t}_{q}) \vee (\underbrace{\neg p \wedge t}_{q}) = (\underbrace{\neg p \wedge t}_{q}).$$

$q \vee q \equiv q$

$\cancel{p \vee p \equiv p}$

$$\neg(\neg p \vee t) \equiv \neg\neg p \wedge \neg t$$