Note Title 10/18/2013

Theorem If $5 \nmid ab$

then $5 \nmid a$ and $5 \nmid b$.

$a \mid b$

a divides b evenly

$a \nmid b$

a does not divide b evenly

Proof Assume it is not true that

$5 \nmid a$ and $5 \nmid b$.

$\neg(5 \nmid a \wedge 5 \nmid b)$

$\Updownarrow$

$\neg 5 \nmid a \vee \neg 5 \nmid b$.

Therefore $5 \mid a$ or $5 \mid b$.

Case 1: $5 \mid a$ evenly.

$a = 5c$ for some integer c.

$ba = 5cb$ therefore $5 \mid ab$.

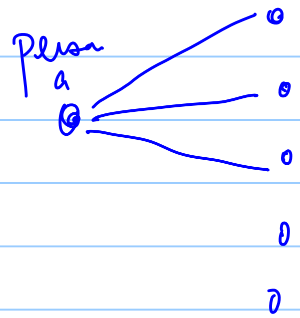
Case 2: $5 \mid b$ evenly.

$b = 5c$ for some int c.

$ba = 5ca$ $\therefore$ $5 \mid ab$.

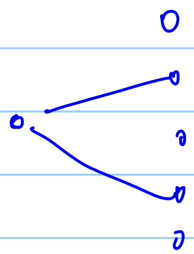
Theorem Among six people there either 3 mutual friends or 3 mutual strangers.

Proof



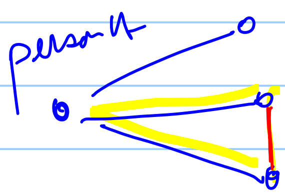
Take person a + look at his relationship to the five other people.

Case A) Either he is friends with at least 3 people



Case B) Or he is strangers with at least 3 people.

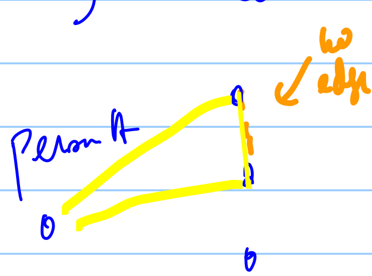
Case A) look 3 friends of person A



if all 3 are mutual strangers, the theorem holds.

if two among the 3 are friends, there are 3 mutual friends.

Case B) look at 3 strangers of A.



if they are mutual friends, the the theorem holds.

if two among them are strangers then there are 3 mutual strangers.

For real # x $$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0. \end{cases}$$

Theorem: For any two real numbers x + y, $|xy| = |x| \cdot |y|$.

Case 1: $x < 0$ and $y < 0$. $|x| \cdot |y| = (-x)(-y) = x \cdot y$.

$|x \cdot y| = x \cdot y$

$xy > 0$ $\therefore |x| \cdot |y| = |xy|$

Case 2: $x \geq 0$ $y < 0$

Case 3: $x < 0$ $y \geq 0$

Case 4: $x \geq 0$, $y \geq 0$.

Set - collection of objects. A, B, C ...

element - object in a set. a, b, c, x, y, z ...

$A = \{2, 4, 6, 8\} = \{8, 6, 2, 4\}$.

$2 \in A$ $5 \notin A$.

Empty set

$\emptyset$

$\{\}$

$\forall a \quad a \notin \emptyset$

$A = \{2, 4, 6, 8, \ldots, 100\}$ even numbers (int) ~~xxxx~~ positive at most 100.

$B = \{2, 4, 6, \ldots\}$

Size of a set (cardinality)

$|A| = 50.$

$D = \{4, 5, 6\}$ $D = E$

$E = \{6, 5, 4\}$ $E \neq F$

$F = \{4, 5, 1, 1\}$ $D \neq F.$