Note Title 10/21/2013

Common Sets in math:

Real numbers $\mathbb{R}$

Integers $\mathbb{Z}$

Natural numbers $\mathbb{N}$. positive integers $\{1, 2, 3, \ldots\}$

Rational numbers $\mathbb{Q}$

$$\mathbb{R}^+ = \{\text{positive real numbers}\}.$$

<u>Set Def</u>

$$A = \{x \in S : P(x)\}$$

predicate.

$$B = \{x \in \mathbb{Z} : x \text{ is even and positive}\}.$$

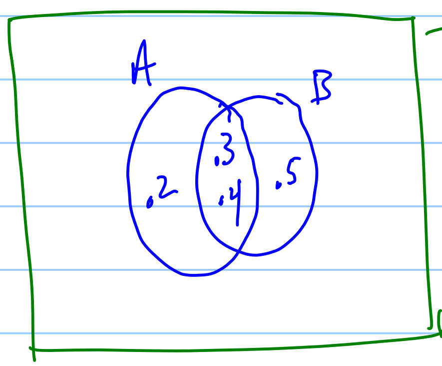
$$C = \{x \in \mathbb{R} : |x| \leq 1\}$$

$$= \{x \in \mathbb{R} : -1 \leq x \leq 1\}.$$

"such that".

Universe Set containing all elements U

<u>Venn Diagram</u>:



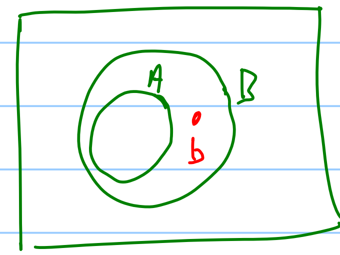
$\to U$ universe.

$A = \{2, 3, 4\}$

$B = \{3, 4, 5\}$

A is a subset of B $A \subseteq B$

$A \subseteq B$ iff $\forall x \, (x \in A \to x \in B)$.

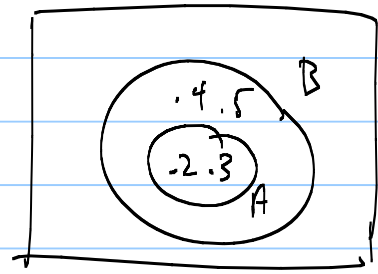


If $A \subseteq B$ then either $A = B$

or there is some element b, $b \in B$ and $b \notin A$.

$B = \{2, 3, 4, 5\}$

$A = \{2, 3\}$.



$A \subseteq B$ $2 \in A$ $2 \in B$

$3 \in A$ $4 \in B$.

$4 \notin A$

If $A \subseteq B$ and $A \neq B$ $A \subset B$

A is a proper subset of B.

$A \subset B \iff A \subseteq B$ and $A \neq B$.

$A = B \iff A \subseteq B$ and $B \subseteq A$.

$A \subseteq B \iff A = B$ or $A \subset B$.

$A = \{x \in \mathbb{Z} : \quad 0 \le x < 13 \quad \text{and} \quad x \text{ is odd}\}$

$B = \{3, 5, 11\}$.

$C = \{1, 3, 5, 7, 9, 11\}$

$D = \{3, 2, 5\}$.

$\cancel{D \not\subset A}$ $\quad 2 \in D$, $2 \notin A$.

$D \not\subseteq A$

$B \subseteq C \qquad B \subset C$

$\left. \begin{array}{l} A \subseteq C \\ C \subseteq A \end{array} \right\} \Rightarrow C = A$.

$S = \{ \{1\}, \{2\}, \{3,2\}, \{1,2\} \}$

$|S| = 4$

$T = \{\emptyset, \{1\}\} \qquad |T| = 2$.

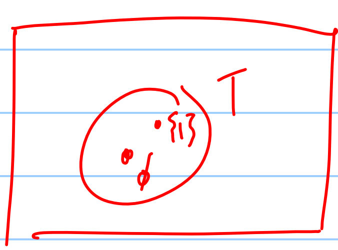
$\underbrace{\{\emptyset\}}_{\text{cardinality } 1} \ne \underbrace{\emptyset}_{\text{cardinality } 0}$.

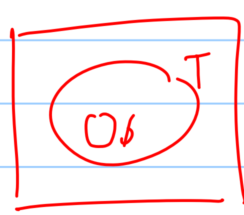
$V = \{1, \{1\}, \{1, 2, 3\}\}$

$|V|$.

$\emptyset \subseteq A$

$\emptyset \in A$





$\emptyset \subseteq$ [illegible]

$S = \{ \boxed{\{1\}}, \boxed{\{2\}}, \boxed{\{3,2\}}, \boxed{\{1,2\}} \}$

$\{1\} \not\subseteq S$?

$\{1\} \in S$ Yes.

$\{ \{1\}, \{2\} \} \subseteq S$

$\{ \boxed{\{1\}} \} \subseteq S$.

Set $A = \{a, b, c\}$ A is finite

Power set of A is the set of all subsets of A

$P(A)$.

$P(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

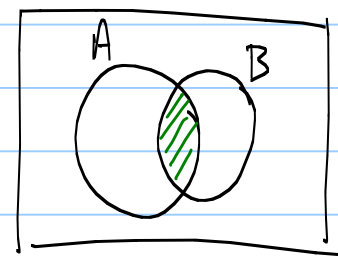
$|P(A)| = 8$.

If $B \subseteq A$ then $\Longleftrightarrow$ $B \in P(A)$

Finite set C and $|C| = n$. $|P(C)| = 2^n$.

Two sets A & B.

$A \cap B = \{x \mid x \in A$ and $x \in B\}$



$A \cap B$.

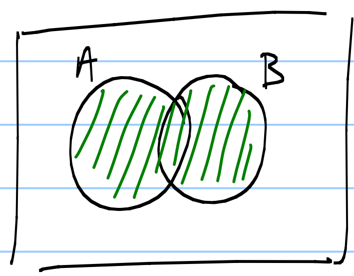
$A = \{x \in \mathbb{Z} : x \text{ is prime}\}$.
$B = \{x \in \mathbb{Z} : x \text{ is even}\}$
$C = \{x \in \mathbb{Z} : 1 < x < 20\}$.

$A \cap B = \{2\}$

$A \cap C = \{2, 3, 5, 7, 11, 13, 17, 19\}$

Union: $A \cup B = \{x : x \in A$ or $x \in B\}$

$D = \{x \in \mathbb{Z} : |x| \leq 10\}$.



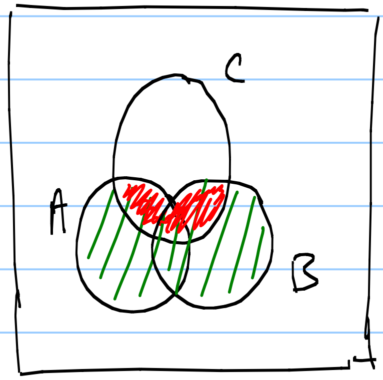
$A \cup B$.

$C \cup D = \{-10, -9, \ldots, 19\}$

$C \cap D = \{2, \ldots, 10\}$.

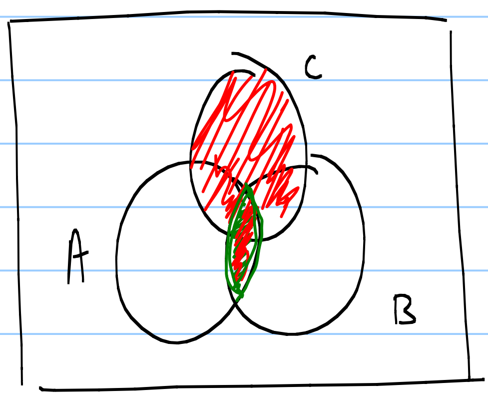
$A \cup B \cap C \overset{?}{=} (A \cup B) \cap C$

or $A \cup (B \cap C)$



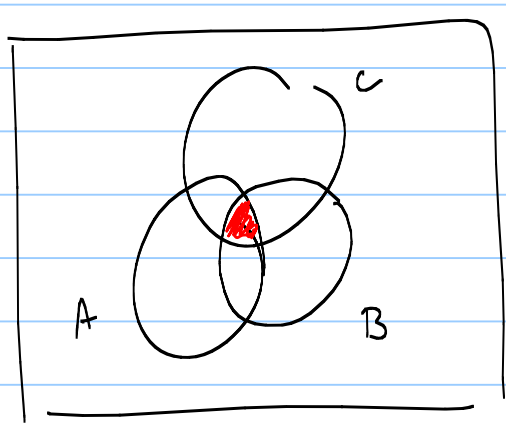
$(A \cup B) \cap C$

$(A \cup B) \cap C$.



$(A \cap B) \cup C$

$A \cap B$



$A \cap B \cap C$

$A \cap B \cap C = \{7\}$.

$(B \cap C) \cup (A \cap D)$

$(A \cap D) = A$

$= \{1, 2, 3, 4, 7, 8, 11\}$.

$A = \{1, 2, 4, 7, 8\}$

$B = \{1, 3, 7, 9, 10, 11\}$

$C = \{x \in \mathbb{Z} : x \text{ prime}\}$

$D = \{x \in \mathbb{Z} : |x| \leq 10\}$

$(B \cap C) \cup A$

$B \cap C = \{3, 7, 11\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 7, 8, 11\}$.

Laws of Sets:

$$A \cap B = B \cap A$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cup C = A \cup (B \cup C).$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$