Relation between sets A & B $\quad R \subseteq A \times B$.

Relation on a set. $\quad R \subseteq A \times A$.

$\overline{A \cup (B \cap C)}$ $\qquad A \cap B \cap (C \overset{?}{\subseteq} D)$

$A \cap B \cap C \subseteq D$

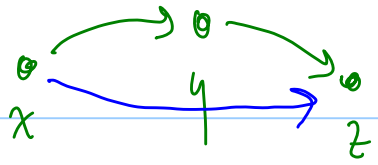
$(A \cap B) \cup C \subseteq D$.

~~$(x \in A) \wedge B$~~

$x \in A \cap B$

Properties of binary relations on a set.

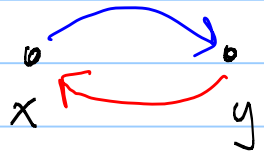
Reflexive	xRx	$\forall x \in A$.	:)
Irreflexive	$x \not R x$	$\forall x \in A$	:(
Transitive.	xRy and $yRz \Rightarrow xRz$		



Example $\mathbb{R}$ $x \leq y$ $x \leq y$ and $y \leq z \Rightarrow x \leq z$.

A ~~relation~~ binary relation R on a set A is symmetric if

$$xRy \Rightarrow yRx.$$



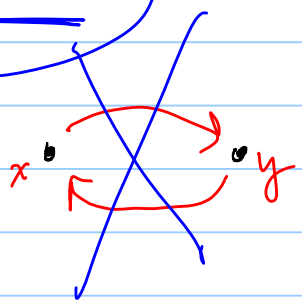
Domain $\mathbb{Z}$ xPy if x is rel prime to y.

then yPx. $\forall P q$

$q P 8$

A binary relation R on a set A is anti-symmetric if:

$$xRy \text{ and } yRx \Rightarrow x = y.$$



OK under anti-symmetric 

OK under anti-symmetric 

Ex: Domain $\mathbb{R}$ xRy iff $x \leq y$.

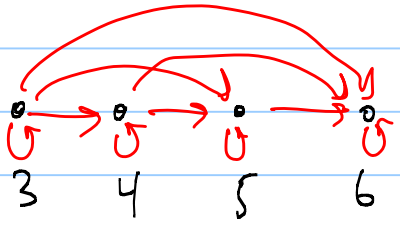
$x \leq y$ and $y \leq x \Rightarrow x = y$.

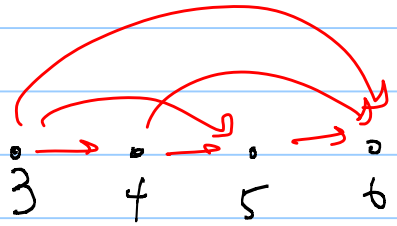
Domain $\mathbb{R}$ xLy $x < y$.

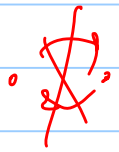
$x < y$ and $y < x$ is never satisfied.

$A = \{3, 4, 5, 6\}$.

$\leq$







Domain $\mathbb{Z}^+$ xPy iff x and y are prime to y.

Reflexive?

~~Reflexive~~?

Is Reflexive? Yes

Transitive? No $\exists 3P7$ $7P9$

$3 \not P 9$.

Symmetric? Yes.

Domain $\mathbb{Z}^+$ xWy y is integer power of x.

$y = x^n$ $n \in \mathbb{Z}^+$

Reflexive (Yes) xWx $x = x^1$

Transitive? Yes. xWy $yWz \Rightarrow xWz$.

$y = x^n$ $z = y^m$ $z = y^m = (x^n)^m = x^{n \cdot m}$

Symmetric? xWy $y = x^n$

yWx $x = y^m$ $\Rightarrow$ $x = y$.

Domain: Set phones in phone network.

xCy if x and y have a call on a given day.

reflexive? a reflexive.

not transitive.

Symmetric

Domain: Set of people. xMy x + y have same mother.

Reflexive xMx.

Transitive: Yes.

Symmetric: Yes.

xPy x + y Share a parent.

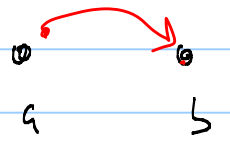
Reflexive: Yes.

Transitive: No. Symmetric: Yes. xPy yPx.

Directed Graph: (digraph) is a pair:

(V, E) V set of objects called Vertices (nodes)
single element of V : vertex.

E set of edges $E \subseteq V \times V$ — E is a relation on set V.



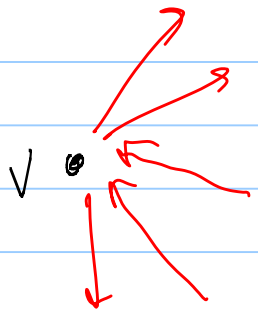
edge (a,b)

a is the tail of the edge
b is the head of the edge.

self-loop : (a,a).

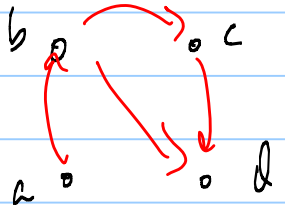
$$\text{in-degree}(v) = |\{u \mid (u,v) \in E\}|$$ # edges pointing in to v.

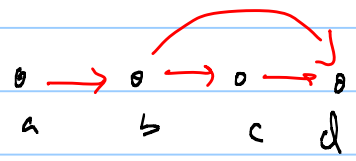
$$\text{out-degree}(v) = |\{u \mid (v,u) \in E\}|$$ # edges pointing out of v.



in-degree(v) = 2
out-degree(v) = 3.

Graph $V = \{a,b,c,d\}$ $E = \{(a,b),(b,c),(c,d),(b,d)\}$





V = { set of all urls }

E = { set of links } (u, v)