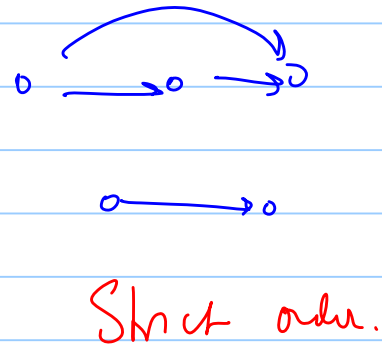
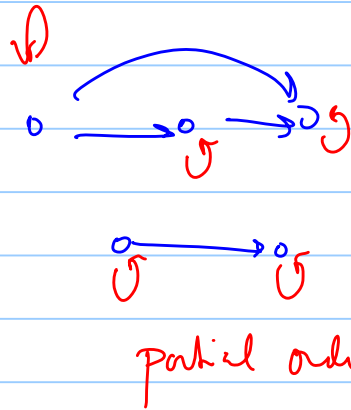


Partial orders $\leq$ Transitive - anti-sym. refl.]
 Strict orders $<$ Transitive anti-reflexive.]



t: transitive
 r: anti-reflexive
 s: Anti-Symmetry.

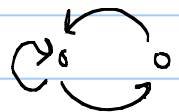
$$(t \wedge r) \rightarrow s$$

$$(\neg s \wedge t) \rightarrow \neg r$$

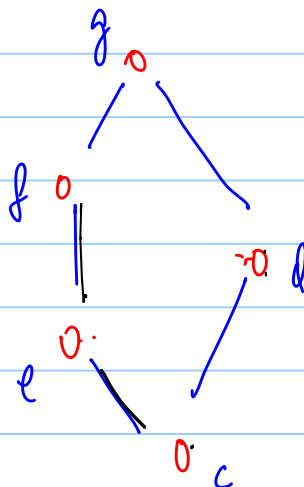
$$\neg s \rightarrow \neg(t \wedge r)$$

$$\neg t \vee \neg r$$

$$t \rightarrow \neg r$$



Strict Order



$c \leq e$

Minimal: c a
 Maximal: g b.

Is comp to d? NO.
 Is comp to e? YES $c \leq e$.

if $x \alpha y$ then x appears lower.

if x appears lower than y then either $x \alpha y$ or incomparable.

Domain $\mathbb{R}$ $(\mathbb{R}, <)$ ^{Can't have}
 $x < y \Rightarrow y \not< x$
[?]
 $x < x$ anti-reflexive

$x < y \wedge y < z \Rightarrow x < z$ transitive.

Anti symmetric : if $x < y \wedge y < x \Rightarrow y = x$

Finite set A . $(P(A), \subset)$ $x < y$
 if $x \subset y$

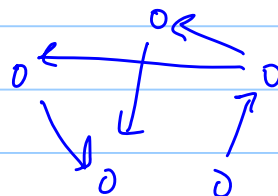
$x \not\subset x$ anti-refl.

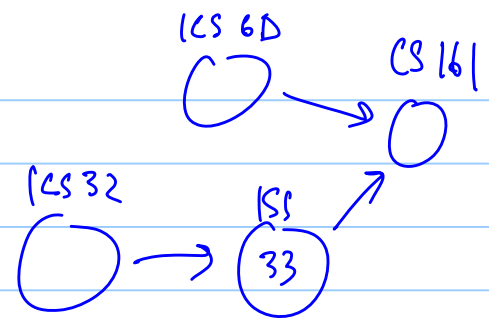
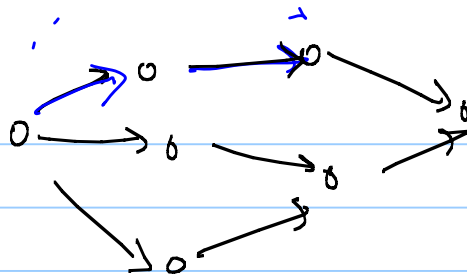
$x \subset y \subset z \Rightarrow x \subset z$ transitive.

If directed graph has no positive length cycles, then it is a directed acyclic graph (DAG)



$\langle 2 \rangle$





Task Diagrams.

~~Nodes~~ Vertices individual tests.
Edges precedence relationships.

If G DAG task diagram.

G^+ gives entire set of constraints.

Thm G directed graph
 G is a DAG iff G^+ is a strict order.

If G has a cycle then G^+ is not a strict order.

$\Rightarrow$

$\langle v_0, v_1, \dots, v_k, v_0 \rangle$
length is k .

Then G^k has edge $\langle v_0, v_0 \rangle$

$(v_0, v_0) \in G^+$

G^+ is not anti-reflexive
and therefore not
a strict order.

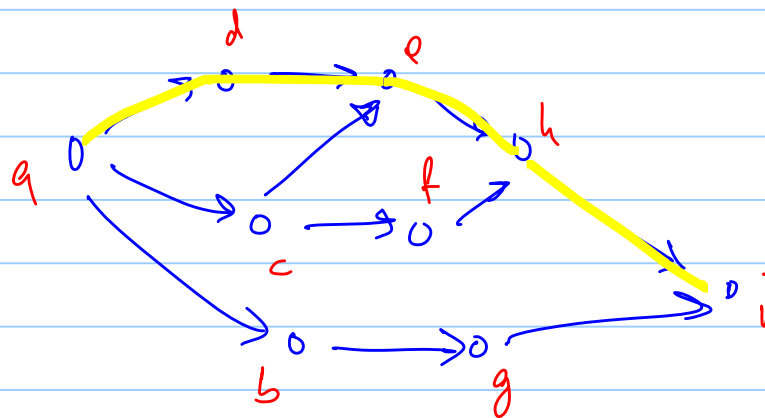
G^+ is always transitive
if G^+ is not a strict order it must have a self-loop.

(a, a) edge in G^+

(a, a) edge in G^k for some k .

then is a path from a to a in G^k
 $\Rightarrow$ cycle.

DAG:

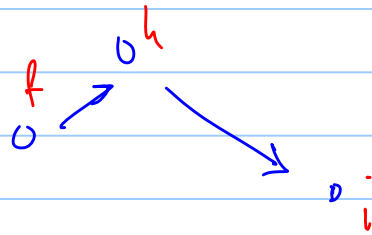


Topological Sort for DAG G : ordering
of vertices if $x \prec y$ then x appears
earlier in ordering.

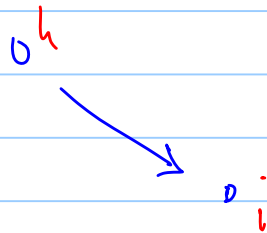
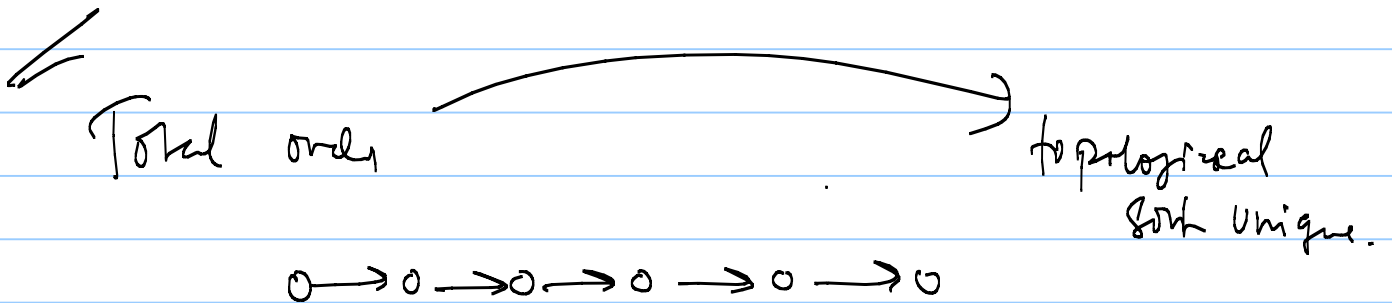
topological
sort

for DAG G

is not unique.



a b c d g e f h i



parallel schedule.

