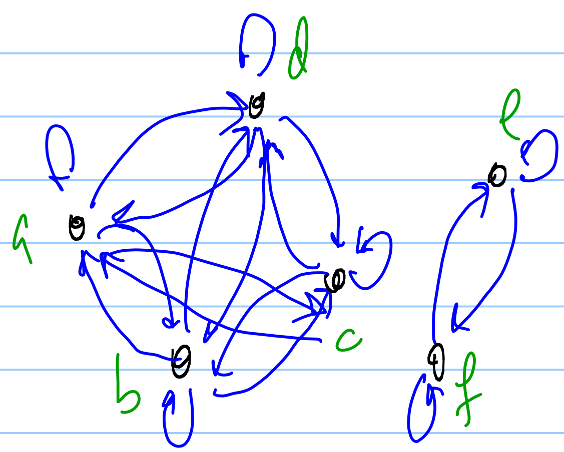
Note Title 11/8/2013

Equivalence Relations:

Binary Relation on a set A
that is: symmetric, transitive, reflexive.

if xRy then $x \sim y$
"x is equivalent to y".



equivalence relation.

6 mod 5 = 1.

Ex

Domain: $\mathbb{Z}$ $x \sim y$ if $x \bmod 5 = y \bmod 5$

reflexive $x \sim x$ $x \bmod 5 = x \bmod 5$ ✓

Symmetric $x \sim y \Rightarrow y \sim x$ $x \bmod 5 = y \bmod 5$

transitive $x \sim y \wedge y \sim z \Rightarrow x \sim z$ $x \bmod 5 = y \bmod 5$ ✓
$y \bmod 5 = z \bmod 5$
then $x \bmod 5 = z \bmod 5$

Domain: set of people $\quad x \sim y$ if x & y have same mother.

refl $\quad x \sim x$

symmetric $\quad x \sim y \Rightarrow y \sim x$.

transitive $\quad x \sim y \wedge y \sim z \Rightarrow x \sim z$.

$x \sim y$ if x and y share a parent. $\quad$ X not an equiv relation.

x & y have same mother

y & z have same father

x & z do not share a parent.

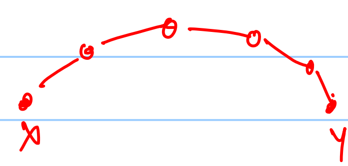
Communication network: vertices = processors

two way communication links.

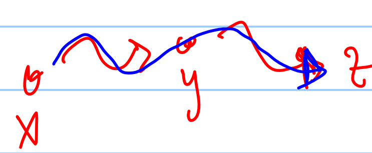
$x \sim y \quad$ if x is reachable from y via a communication path.

$x \sim x \quad$ reflexive.

$x \sim y \Rightarrow y \sim x$



$x \sim y \wedge y \sim z \Rightarrow x \sim z$.



if X is in the domain

$$[x] = \{y \mid x \sim y\}$$

Claim: A) if $x \sim y$ then $[x] = [y]$
B) if $x \not\sim y$ then $[x] \cap [y] = \phi$

A) Suppose $x \sim y$ then any $z \in [x]$ is also in $[y]$

$x \sim z$ (because $z \in [x]$)
$x \sim y = y \sim x$
$\Rightarrow y \sim z$ and $z \in [y]$.

B) Proof by contrapositive
Suppose $\exists z \in [x] \cap [y]$ (i.e. $[x] \cap [y] \neq \phi$)

$z \sim x$ and $z \sim y$
$x \sim z \wedge z \sim y \Rightarrow x \sim y. = !(x \not\sim y)$

$[x] = [y]$

The set of all distinct equivalence classes forms a partition of the domain.

The Domain: $\mathbb{Z}$. $x \sim y$ if $x \bmod [5] = y \bmod [5]$

Equiv classes: $[0]$ $[1]$ $[\cancel{2}]$ $[4]$ $[3]$
$[7]$