$f: X \to Y$ $\quad$ f is 1-1 if $x \neq x' \Rightarrow f(x) \neq f(x')$

onto if $\forall y \in Y \ \exists x \in X$ $f(x) = y$.

$f: \mathbb{Z} \to \mathbb{Z}$ $\quad$ $f(n) = \lceil n/2 \rceil$ $\quad$ $f(5) = \lceil 5/2 \rceil = 3$

not 1-1.

$f(6) = 3$.

$f: \mathbb{Z} \to \mathbb{Z}$ $\quad$ $f(n) = \lceil n \rceil$ $\quad$ $f(2n) = n$.

$f(n) = n$.

$f: \mathbb{R} \to \mathbb{R}$

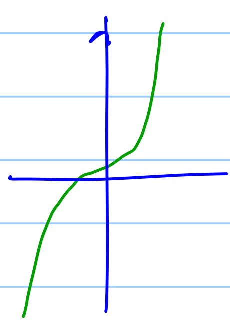
$f(n) = 2n-1$ $\Big\}$ 1-1 & onto.

$f: \mathbb{R} \to \mathbb{R}$

$f(n) = n^2+1$ $\quad$ $f(1) = f(-1)$ not 1-1

$\nexists\, x \in \mathbb{R}$ $f(x) < 0$.

$f(n) = n^3$



1-1 & onto.

$f: X \to Y$

Inverse of f (if it exists) denoted f^{-1}

$$f^{-1} = \{(y,x) : (x,y) \in f\}.$$

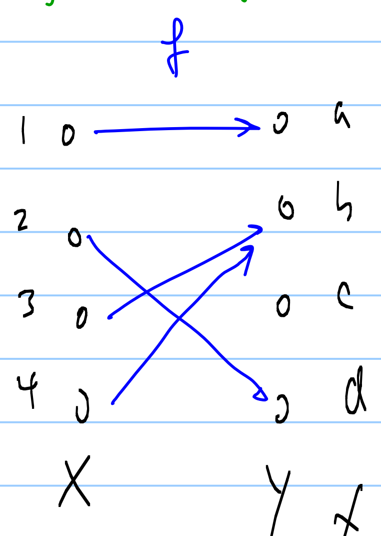
$$f^{-1}: Y \to X.$$

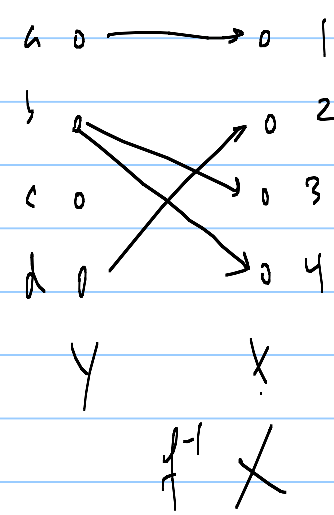
f^{-1} does not always define a function.

f^{-1} defined for every $y \in Y$? (if f is onto)

f^{-1} has to have a unique value for every $y \in Y$? (f is one to one)

A function f has an inverse iff f is a bijection.





$f(x) = 2x+3$ $\qquad$ $f^{-1}(y) = \frac{y-3}{2}$.

$y = 2x+3$

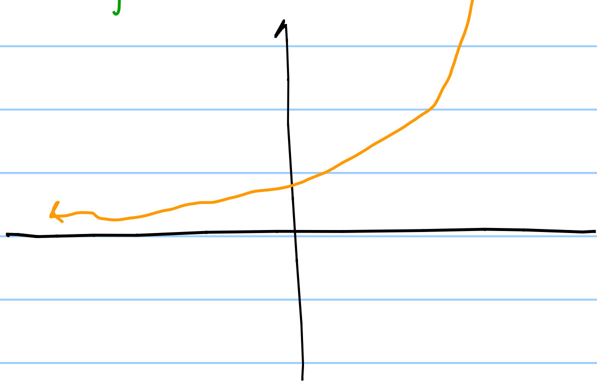
$\frac{y-3}{2} = \cancel{2}x$

$f: \mathbb{R}^+ \to \mathbb{R}^+$.

$f(x) = x^2$ $\qquad$ $f^{-1}(x) = \sqrt{x}$

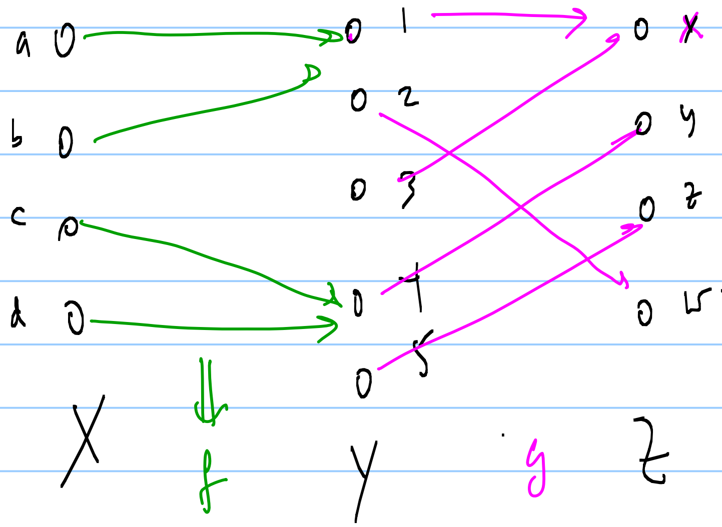
$f: \mathbb{R} \to \mathbb{R}^+$

$f(t) = 3 \cdot 4^t$ $\qquad$ $f^{-1}(n) = \log_4(n/3)$



$f: X \to Y$ $\qquad$ $g: Y \to Z$ $\qquad$ $g(f(x))$

$f: X \to Y$ $\quad g: Y \to Z$ $\quad g(f(x))$



Composition of f & g.

$g \circ f : X \to Z$.

$g \circ f(a) = x$

$g \circ f(c) = y$.

$f : \mathbb{R}^+ \to \mathbb{R}^+ \quad f(x) = x^3 \quad g \circ f = \log_2(x^3)$

$g : \mathbb{R}^+ \to \mathbb{R}^+ \quad g(x) = \log_2 x \quad f \circ g = (\log_2 x)^3$

$\Downarrow$

$\log_2^3 x$

What is $f \circ f^{-1}$? $\quad f \circ f^{-1} = I$.

Identity function on a set A $\quad I_A : A \to A$

$\forall a \in A. \quad I_A(a) = a.$

$f: A \to B$.

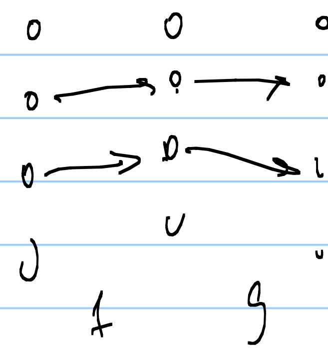
$$f^{-1} \circ f = I_A$$

$$f \circ f^{-1} = I_B.$$

If f & g are 1-1 functions.

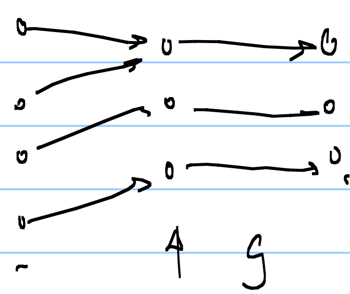
Is $g \circ f$ 1-1?

YES.



If f & g are onto is $g \circ f$?

YES.



If f & g are bijections, then so is $g \circ f$.

Sequence: function maps a consecutive set of integers to some target set

$g(\underline{1}) = 3.67 \quad g(\underline{2}) = 2.88 \quad g(\underline{3}) = 3.15 \quad g(\underline{4}) = 3.75$

$g_{\underline{1}} = 3.67 \quad g_{\underline{2}} = 2.88 \quad g_{\underline{3}} = 3.15 \quad g_{\underline{4}} = 3.75$ "~~[illegible]~~ indices.

3.67, 2.88, 3.15, 3.75

$\{g_n\}$ entire sequence.

g_n element with index n

Infinite sequence : $a_0, a_1, a_2, \ldots$

$b_1, b_2, b_3, \ldots$

$a_n = 2^n$ $\{a_n\}$: $2, 4, 8, 16, \ldots$

Geometric sequences: $\longrightarrow$ ratio between two consecutive #'s is the same.

$\Downarrow$

Common ratio.

$5, 10, 20, 40, 80, \ldots$

$\times 2$

Initial term: a.
Common ratio: r.

$\{g_n\} = a, ar, ar^2, ar^3, \ldots$

Start w/ index of 0 $g_0 = a$ $g_n = ar^n$.

$g_1 = ar$

$\vdots$

$a_0 = 1000 \qquad a_1 = 1000 \cdot (1.03) \qquad a_2 = 1000 \cdot (1.03)^2$

$$\underbrace{1000}_{\text{initial value}} \; (\underbrace{1 + .03}_{\text{common ratio}})^n$$