Note Title 9/30/2013

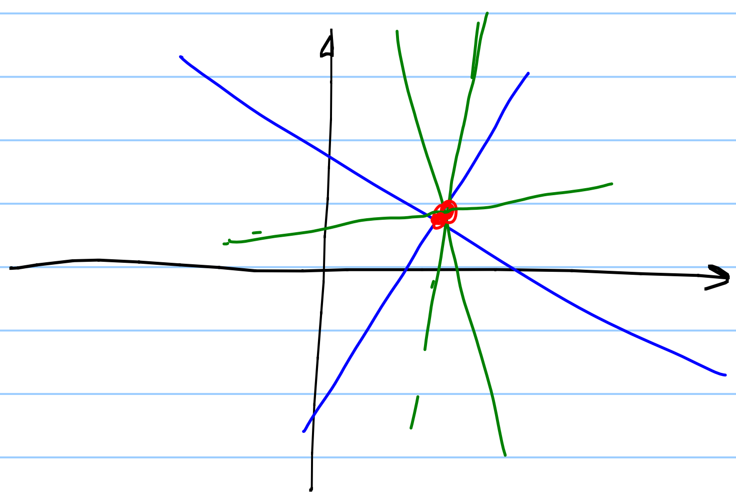
Please put HW for each section on a separate sheet of paper.
A section may get postponed to the following week. That way, you can just add it to the following week's work.

A system of linear equations has either:

inconsistent → (1) No solution.

Consistent { (2) One solution.
(3) Infinite # of solns.

Two systems of linear eqns are equivalent if they have the same set of solutions.



$$\begin{aligned} 1\cdot x_1 + 0x_2 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ 0\cdot x_1 + x_2 + 5x_3 &= -2 \end{aligned}$$

augmented matrix

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

Coefficient matrix

$$\begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix}$$

Matrix $m \times n$ # columns, # rows

System of linear eqns n variables
m equations.

Coeff matrix is $m \times n$.
Augmented matrix is $m \times (n+1)$

Solving a system of linear eqns:

Elementary row operations:

After each row op, system is equivalent to system before.

Basic Row Ops:

Replacement: Replace a row (eqn) with the sum of itself and a multiple of another row. (eqn).

Interchange: Swap two rows.
Scaling: Multiply all entries in a row times a non-zero constant.

$$\begin{array}{cccc} -2 & 0 & 6 & -16 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 \; -1 & 15 \; -\frac{15}{2} & -9 \; \frac{9}{2} \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 0 & -5/2 & +5/2 \end{bmatrix} \rightarrow \begin{array}{l} x_1 \qquad\quad - 3x_3 = 8 \\ 2x_2 + 15x_3 = -9 \\ \qquad -5/2\, x_3 = -5/2 . \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & 1 & -1 \\ & & -15/2 & 15/2 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ & & 3 & -3 \end{array}$$

$$\left.\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right\} \Rightarrow \begin{array}{l} x_1 = 5 \\ x_2 = 3 \\ x_3 = -1 \end{array}$$

Two matrices are "row equivalent" if one can be obtained from the other by a series of elementary row ops.

Two $\underline{\text{matrices}}$ are row equiv $\Rightarrow$ corresponding linear systems are equiv.

Augmented matrices.

The "leading entry" in a row is the left-most non-zero entry.

$$0 \quad 0 \quad -2 \quad 3 \quad 0 \quad 4$$

leading entry.

A matrix is in Row Echelon form if

1. Non-zero rows above zero rows.
2. Each leading entry is to the right of the leading entry of the rows above.
3. All entries in the column below a leading entry are 0.

$$\begin{bmatrix} \square & \times & \times & \times & \times & \times & \times & \times \\ 0 & 0 & 0 & \square & \times & \times & \times & \times \\ 0 & 0 & 0 & 0 & \square & \times & \times & \times \\ 0 & 0 & 0 & 0 & 0 & 0 & \square & \times \end{bmatrix}$$

$\square$ leading entries.
non-zero.

Reduced Row Echelon Form:

(1) Row Echelon Form
(2) Leading entries are all 1.
(3) Leading entries are the only non zero entries in their columns.

$$\begin{bmatrix} 1 & x & x & 0 & 0 & x & 0 & x \\ 0 & 0 & 0 & 1 & 0 & x & 0 & x \\ 0 & 0 & 0 & 0 & 1 & x & 0 & x \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & x \end{bmatrix}$$

Matrix A $\xrightarrow{\text{row ops}}$ Echelon form for A $\xrightarrow{\text{row ops}}$ Red Row Echelon Form.

Algorithm: Gaussian Elimination.

Part I Top to Bottom, covering part of the matrix at top.

① Pick the leftmost col w/ non-zero entry.

new pivot

$$\begin{bmatrix} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{bmatrix}$$

pivot column.

② Pick a non-zero entry in pivot column.
Swap rows so pivot at top.

$$\begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{bmatrix}$$

③ Use row replacement to zero out the entries below pivot.

④ Cover row w/ pivot.

$$\begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{bmatrix}$$

1 ½ -4

$$\begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & 0 & -1/2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & 0 & -1/2 & -1 \end{bmatrix}$$

$$\begin{matrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 1/2 & -4 \\ 0 & 0 & 1 & 2 \end{matrix}$$