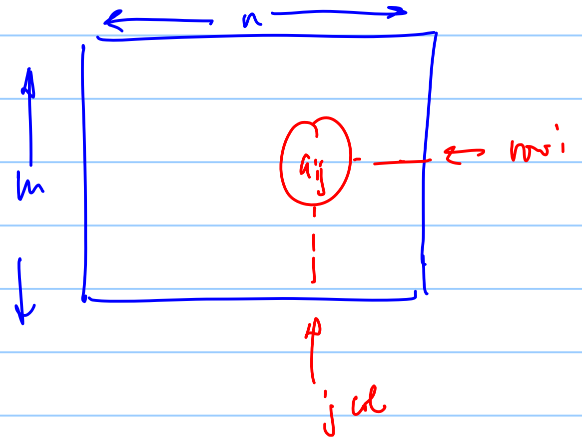
A $m \times n$ matrix

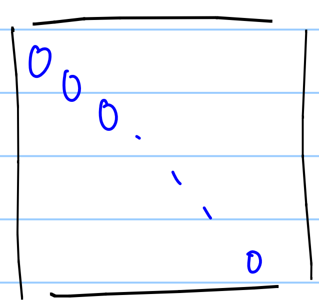


a_{ij} entry row i col j.

(i,j)-entry of A.

$$A = [\vec{a}_1 \; \cdots \; \vec{a}_n] \qquad \vec{a}_i \in \mathbb{R}^n.$$

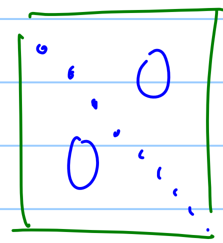
Diagonal entries a_{11} a_{22} a_{33} . . .



main diagonal of matrix

Diagonal matrix $n \times n$ square matrix

all non-diag entries are 0



$$I_n = \begin{bmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ 0 & & & & & 1 \end{bmatrix}$$

0 matrix all entries 0.

To add A & B, A & B must same # rows & cols.

Otherwise ~~##~~ ~~is~~ addition is undefined.

$$\cancel{\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 5 & -1 \\ 2 & 6 & -1 \end{bmatrix}}$$ not defined.

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}$$

r scalar $\quad rA \quad A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

$$3A = \begin{bmatrix} 3 & 9 \\ 6 & 3 \end{bmatrix}$$

$$A - B = A + (-1)\cdot B = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

assuming A, B & C have same dims.

$2A + 3B =$

$A + B = B + A$

$(A+B) + C = A + (B+C)$

$A + 0 = A$

$1 \cdot A = A$

$r(A+B) = rA + rB$

$(r+s)A = rA + sA$

$r(sA) = (rs)A.$

$A\vec{x}$ $\quad$ A is $m \times n$ matrix w/ cols $\vec{a}_1 \cdots \vec{a}_n$

$\vec{x} \in \mathbb{R}^n$

$$A\vec{x} = x_1 \vec{a}_1 + \cdots + x_n \vec{a}_n.$$

A $n \times m$ matrix $\qquad\qquad$ B $l \times m$ matrix

$T : \mathbb{R}^n \to \mathbb{R}^m \qquad\qquad Q : \mathbb{R}^m \to \mathbb{R}^l$

$$\begin{tikzcd}
\vec{x} \in \mathbb{R}^n \arrow[r, bend left, "T"] \arrow[rr, bend right=60, "(BA)\vec{x}"'] & T(\vec{x}) \in \mathbb{R}^m \arrow[r, bend left, "Q"] & Q(T(\vec{x})) \in \mathbb{R}^l
\end{tikzcd}$$

$\qquad\qquad [A\vec{x}] \qquad\qquad B[A\vec{x}].$

$$A\vec{x} = x_1 \vec{a}_1 + \cdots + x_n \vec{a}_n$$

$$B A\vec{x} = B \underbrace{x_1 \vec{a}_1} + \cdots + B \underbrace{x_n \vec{a}_n}$$

$$x_1 B\vec{a}_1 + \cdots + x_n B\vec{a}_n$$

$$= \underbrace{\begin{bmatrix} B\vec{a}_1 & \cdots & B\vec{a}_n \end{bmatrix}}_{(B\cdot A)} \cdot \vec{x}$$

cols: n

rows: l.

B : $l \times m$ matrix

A $m \times n$ matrix

$$\underbrace{B \cdot A}_{l \times n.} = \begin{bmatrix} B\vec{a}_1 & B\vec{a}_2 & B\vec{a}_n \end{bmatrix}$$

only defined if

cols of B = # rows of A.

$$\underbrace{\begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}}_{B} \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} B \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} & B \begin{bmatrix} -1 \\ 0 \end{bmatrix} & B \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

4×2 $\quad$ 2×3 $\quad$ $4 \times 3.$

$$= \begin{bmatrix} 0 & -2 & -1 \\ 3 & -3 & 0 \\ 6 & -4 & 1 \\ 5 & -1 & 2 \end{bmatrix}$$

Every col in product is a lin comb of cols in first matrix.

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 4 & 3 \end{bmatrix}$$

$A \cdot B$ is $\underline{2}$ by $\underline{3}$

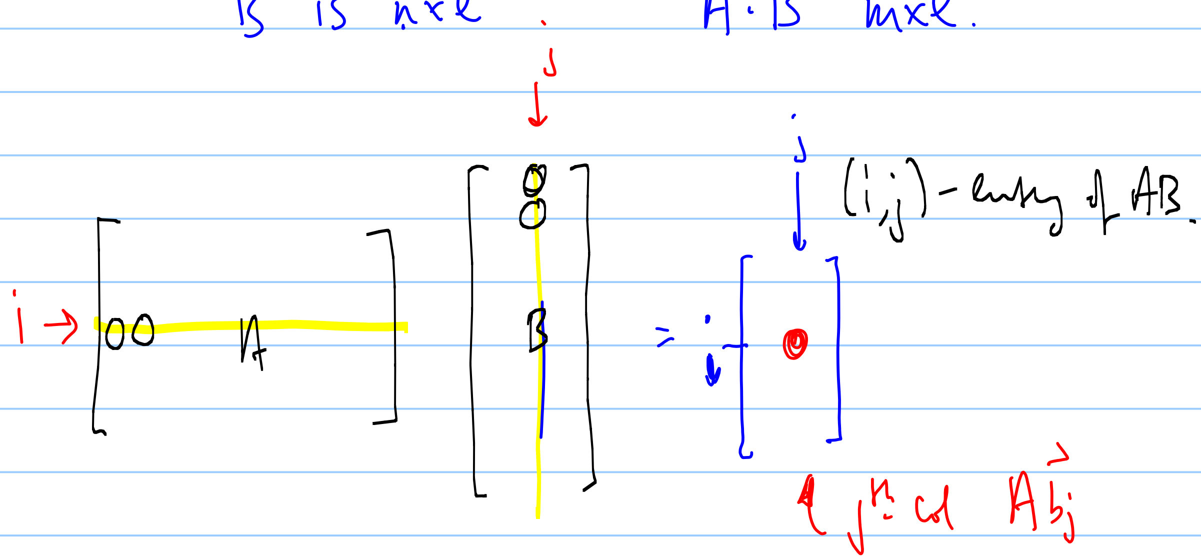
$AB = \begin{bmatrix} & & \boxed{} \end{bmatrix}$ (2,3)-entry of AB.

$\rightarrow -2 \cdot 1 + 4 \cdot 3$

$A\vec{b}_3$

In general: A is $m \times n$

B is $n \times l$. $A \cdot B$ $m \times l$.



(i,j)-entry is: $a_{i1} \cdot b_{1j} + a_{i2} \cdot b_{2j} + \ldots + a_{in} \cdot b_{nj}$

$$A(BC) = (AB)C$$

$$A(B+C) = AB + AC$$

$$rAB = ArB.$$

A is $m \times n$

$$I_m \cdot A = A$$

$$\begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$