Section 4.1 Vector Spaces.

Note Title 10/23/2013

$\mathbb{R}^n$. =

Definition: A **vector space** V is a non-empty set of objects call vectors, with two operations: vector addition

for now real number. → scalar multiplication. They obey 10 rules.

$\vec{u}, \vec{v}, \vec{w} \in V$ Scalars c + d.

1. Closed under addition: $\vec{u}+\vec{v} \in V$.
2. $\vec{u}+\vec{v} = \vec{v}+\vec{u}$.
3. $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$.
4. There is a $\vec{0} \in V$ $\quad \vec{u}+\vec{0}=\vec{u}$.
5. Additive inverse. For every $\vec{u}$, there is a $(-\vec{u})$ $\quad \vec{u}+(-\vec{u})=\vec{0}$.
6. Closed under scalar mult. $c\vec{u} \in V$.
7. $c(\vec{u}+\vec{v}) = c\vec{u}+c\vec{v}$
8. $(c+d)\vec{u} = c\vec{u}+d\vec{u}$.
9. $c(d\vec{u}) = (cd)\vec{u}$.
10. scalar 1: $1\cdot\vec{u} = \vec{u}$.

Example: $\{\ldots, y_{-2}, y_{-1}, y_0, y_1, y_2, y_3 \ldots\}$ $\quad y_i \in \mathbb{R}$.

Example 2: polynomials of degree $\leq n$.

$$p(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n$$

$z(t) = 0.$

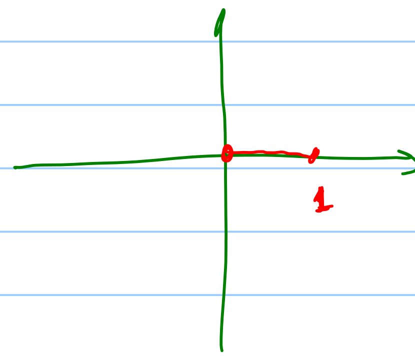
$\Rightarrow$ Real valued functions over a set D.

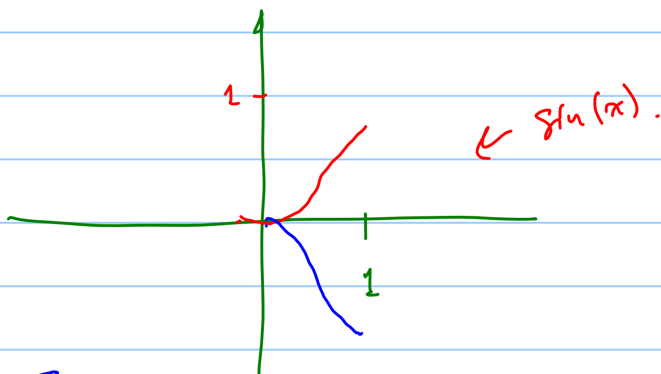
V Real valued functions defined on the interval $[0,1]$.

$\sin(x)$ $\qquad$ $f(x) = 0.$

x^2

$-\sin(x).$ $\qquad$ $2x+3.$ $\qquad$ $g(x) + f(x) = g(x)$





Subspace H of a Vector Space V. $\qquad H \subseteq V.$

1. $\vec{0} \in H$
2. Closed under addition: $\vec{u}, \vec{v} \in H$ then $\vec{u} + \vec{v} \in H$.
3. $\vec{u} \in H \qquad c\vec{u} \in H.$

$H = \{\vec{0}\}.$ $\qquad \{\vec{0}\} \subseteq H \subseteq V.$

Example   V: real-valued functions over $\mathbb{R}$.

H: polynomials of degree $\leq n$.

H is a subspace of V.

Question   Is $\mathbb{R}^2$ a subspace of $\mathbb{R}^3$?

(x, y) (x, y, z)

$\mathbb{R}^2$ is not a subset of $\mathbb{R}^3$.

$$H = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} ; \ x \in \mathbb{R} \ y \in \mathbb{R} \right\}. \qquad H \subseteq \mathbb{R}^3$$

$(x, y, 0)$.

H is a subspace of $\mathbb{R}^3$.

Vector space V. $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \in V$.

Span $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$ is subspace of V.

For example: $\vec{v}_1, \vec{v}_2 \in V$,   Span $\{\vec{v}_1, \vec{v}_2\}$

Verify ① $\vec{0} \in \text{Span}\{\vec{v}_1, \vec{v}_2\}$.

② $\text{Span}\{\vec{v}_1, \vec{v}_2\}$ is closed under add

③ " under scalar mult.

① $0 \cdot \vec{v}_1 + 0 \cdot \vec{v}_2 = \vec{0} \in \text{Span}\{\vec{v}_1, \vec{v}_2\}$.

② $\vec{u} = a\vec{v}_1 + b\vec{v}_2$ $\quad \vec{u} + \vec{w} = a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_1 + d\vec{v}_2$

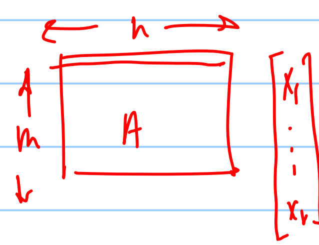
$\vec{w} = c\vec{v}_1 + d\vec{v}_2$

$= (a+c)\vec{v}_1 + (b+d)\vec{v}_2 \in \text{Span}\{\vec{v}_1, \vec{v}_2\}$

③ $c\vec{u} = c(a\vec{v}_1 + b\vec{v}_2) = ac\vec{v}_1 + cb\vec{v}_2 \in \text{Span}\{\vec{v}_1, \vec{v}_2\}$.

✓

Section 4.2



Def Let A be an $m \times n$ matrix.
The null space of A ($\text{nul } A$) is the set of all solutions to $A\vec{x} = \vec{0}$.

$$\text{nul } A \subseteq \mathbb{R}^n$$

① $\vec{0} \in \text{nul } A$. ✓ $\quad A \cdot \vec{0} = \vec{0}$

② If $A\vec{u} = 0$ and $A\vec{v} = \vec{0}$ then $A(\vec{u} + \vec{v}) = \vec{0}$

$\vec{u} \in \text{nul } A$ + $\vec{v} \in \text{nul } A$ $\quad \vec{u} + \vec{v} \in \text{nul } A$

$$A(\vec{u}+\vec{v}) = A\vec{u} + A\vec{v} = \vec{0} + \vec{0} = \vec{0}.$$

(3) If $\vec{u} \in \text{nul} A$ then $c\vec{u} \in \text{nul} A$

If $\underline{A\vec{u}} = \vec{0}$ then $A(c\vec{u}) = \vec{0}$

$$\downarrow$$

$$cA\vec{u} = c\vec{0} = \vec{0}.$$

Example: Solutions to

$$x + 2y - 3z = w \quad \checkmark$$
$$2x - y + 5w = z.$$

$(w, x, y, z) \in \mathbb{R}^4$ Set of Solutions $\subseteq \mathbb{R}^4$.

$$-w + x + 2y - 3z = 0$$
$$5w + 2x - y - z = 0.$$

$$\begin{bmatrix} -1 & 1 & 2 & -3 \\ 5 & -4 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 2 & -3 \\ 0 & 1 & 9 & -16 \end{bmatrix}$$

$$\begin{bmatrix} +1 & 0 & +7 & -13 \\ 0 & 1 & 9 & -16 \end{bmatrix}$$

$$w + 7y - 13z = 0.$$
$$x + 9y - 16z = 0.$$

$$w = -7y + 13z$$
$$x = -9y + 16z.$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7y + 13z \\ -9y + 16z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} -7 \\ -9 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 13 \\ +16 \\ 0 \\ 1 \end{bmatrix}$$