Note Title 10/25/2013

$$T(\vec{x}) = A\vec{x}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & a \\ 0 & 1 & 3 & 5 & b \\ \underbrace{0 \quad 0 \quad 0 \quad 0}_{} & & & & c \end{array}\right]$$

Solutions to: $\begin{aligned} x + 2y - 3z &= w \\ 2x - y + 5w &= z. \end{aligned}$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\begin{bmatrix} -1 & 1 & 2 & -3 \\ 5 & 2 & -1 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 7 & -19 \\ 0 & 1 & 9 & -16 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 & -19 \\ 0 & 1 & 9 & -16 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \vec{0}$$

$$w = -7y + 19z$$
$$x = -9y + 16z.$$

$$\begin{bmatrix} -7y + 19z \\ -9y + 16z \\ y \\ z \end{bmatrix} \rightsquigarrow y \begin{bmatrix} -7 \\ -9 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 19 \\ 16 \\ 0 \\ 1 \end{bmatrix}$$

5 variables : $x_2\ x_3\ x_5$

$$x_2 \begin{bmatrix} * \\ 1 \\ 0 \\ * \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} * \\ 0 \\ 1 \\ * \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} * \\ 0 \\ 0 \\ * \\ 1 \end{bmatrix}$$

free variables = # vectors in lin comb.

vectors are lin. indep.

Column space of a matrix:

The column space of an $m \times n$ matrix A (Col A) is the span of its columns.

$A = [\vec{a}_1 \ \cdots \ \vec{a}_n]$ then $\text{Col}(A) = \text{Span}\{\vec{a}_1, \cdots \vec{a}_n\}$.

subspace of $\mathbb{R}^m$

$\text{Col } A = \{\vec{b} : \vec{b} \in \mathbb{R}^m$ and $A\vec{x} = \vec{b}$ is consistent$\}$.

$$A = \begin{bmatrix} 2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

Nul(A) subspace of $\mathbb{R}^4$

Col(A) subspace of $\mathbb{R}^3$

Find non-zero vec in Col(A) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

Find non-zero vec in Nul(A) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

Is $\begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$ in Nul(A)? No.

$$\begin{bmatrix} 2 & 1 & 0 & 2 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

Is $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$ in Col(A)? Yes

Linear transformation $T : V \to W$. (V + W are vector spaces).

$T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$. $\vec{u}, \vec{v} \in V$.

$T(c\vec{u}) = cT(\vec{u})$. scalar c.

Kernel (T) = Null Space (T) $\vec{u} \in V$ $T(\vec{u}) = \vec{0}$.

Range (T) all $\vec{b} \in W$ s.t. there is a $\vec{v} \in V$,

$$T(\vec{v}) = \vec{b}.$$