Note Title 11/1/2013

Example for Sec 4.4

$\mathbb{P}_2$ all polys degree ≤ 2. $\{1, t, t^2\}$.

$\Rightarrow \mathcal{B} = \{ 1+t^2,\ 1+t,\ t+t^2 \}$

$$\begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix}$$

express $p(t) = 6+3t-t^2$ relative $\mathcal{B}$

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{b}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$c_1 \vec{b}_1 + c_2 \vec{b}_2 + c_3 \vec{b}_3 = \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}$$

Aug:
$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 3 \\ 1 & 1 & 0 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$c_1 = 1$
$c_2 = 5$
$c_3 = -2$

$$1 \cdot (t^2+1) + 5(1+t) - 2(t+t^2) = 6+3t-t^2$$

$$\begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

Section 4.5 Dimension of a Vector Space.

$\Rightarrow$ If a vector space V has basis $\mathcal{B} = \{\vec{b}_1, \dots, \vec{b}_n\}$ then any set of vectors in V with ~~the~~ more than n vectors is linearly dependent.

Pf $\quad \vec{u}_1 \cdots \vec{u}_p \quad p > n. \quad \vec{u}_i \in V.$

Coordinate vectors $\quad [\vec{u}_i]_{\mathcal{B}} \in \mathbb{R}^n.$

$n \times p$ matrix $\quad \begin{bmatrix} & \text{col} & \end{bmatrix}$ $\quad$ If $p > n$ cols are lin dep.

$\vec{u}_1 \cdots \vec{u}_p$ lin dep.

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If $\mathcal{B} = \{\vec{b}_1 \dots \vec{b}_n\}$ basis for V.
Any basis for V has $\geq n$ vectors.

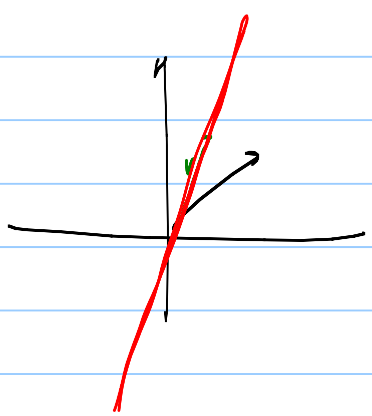
Suppose $\vec{b}_1' \cdots \vec{b}_{n-1}$ basis $\Rightarrow$ lin indep.

Def If vector space V spanned by a finite set then V is finite-dimensional.
The dimension of V ($\dim V$) is the number of vectors in a basis for V.

The dimension of $\{\vec{0}\}$ is 0.
If V is not spanned by a finite set it is infinite-dimensional.

Dimension of $\mathbb{P}_3$ (poly's of degree ≤ 3)?
↳ 4 Basis: $\{1, t, t^2, t^3\}$.

Subspaces of $\mathbb{R}^3$. $\{\vec{0}\}$ $\dim = 0$.



Span $\{\vec{v}\}$

Span $\{\vec{v}_1, \vec{v}_2\}$ $\vec{v}_1 \neq c\vec{v}_2$
$\vec{v}_1 \neq \vec{0}$ $\vec{v}_2 \neq \vec{0}$.

↳ plane through origin

Span $\{\vec{v}_1\ \vec{v}_2\ \vec{v}_3\}$
$\vec{v}_3$ not in Span $\{\vec{v}_1\ \vec{v}_2\}$
↳ dim 3 $= \mathbb{R}^3$.

H is subspace of a finite dimensional vector space V.

Any lin indep set in H can be expanded to a basis for H.

$$\dim H \leq \dim V.$$

$\{\vec{u}_1, \ldots, \vec{u}_k\}$ lin indep.

$$\operatorname{Span}\{\vec{u}_1, \ldots, \vec{u}_k\} \underset{\subseteq}{\neq} H.$$