Note Title 11/8/2013

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

Det:

algebraic function

of $a_{ij}$'s $\neq 0$

iff $A$ is invertible.

$A, B$ $n \times n$

$$\det(AB) = \det(A)\det(B).$$

$$\det(A+B) \neq \det(A) + \det(B).$$

$$A\vec{x} = \vec{b} \qquad x_i = \frac{\det A_i(\vec{b})}{\det(A)}$$

$A$ w/ $i^{th}$ col replaced by $\vec{b}$.

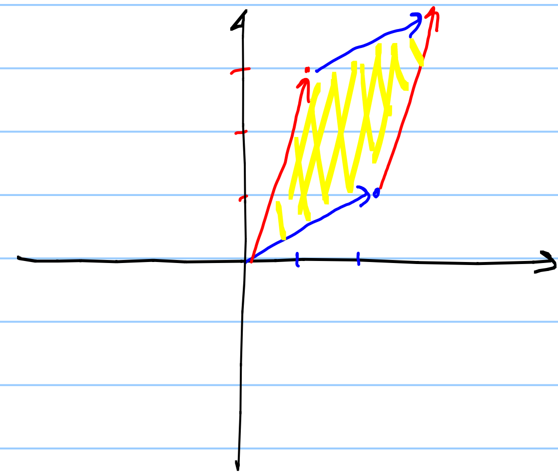
$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix}$$

$$C_{ij} = (-1)^{i+j} \det(A_{ij})$$

matrix $A$ w/ row $i$ + col $j$ deleted.

If $A$ is $2\times 2$ matrix the area of the parallelogram defined by the cols of $A$ is $|\det A|$.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

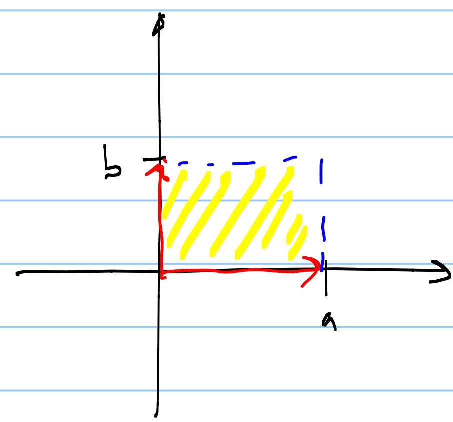


$$|\det A| = 6 - 1 = 5.$$

If $A$ is $3\times 3$ matrix the ~~area~~ volume of the parallelipiped defined by cols of $A$ is $|\det A|$.

Special Case:

$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$




$$|\det(A)| = |ab|$$

Fact: $\det(A) = \det(A^T)$.

$$A \xrightarrow[\text{col swap}]{\text{col repl}} D \ (\text{diagonal})$$

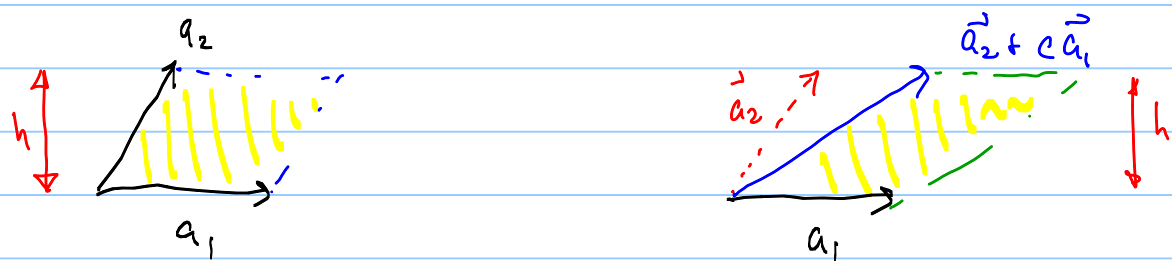
$$|\det A| = |\det D|.$$

$\Downarrow$ parallelepiped.

$\Downarrow$ rectangle whose area $= |\det D|$.

Column swap doesn't change the parallelepiped.

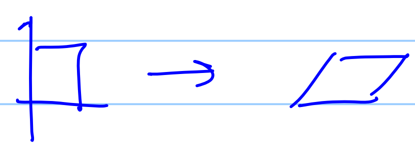
$$A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 \end{bmatrix} \longrightarrow \begin{bmatrix} \vec{a}_1 & \vec{a}_2 + c\vec{a}_1 \end{bmatrix}$$




Area $= |a_1| \cdot (\text{height } h)$ $\qquad$ $|a_1| \cdot h =$ area.

**Theorem** $\quad T : \mathbb{R}^2 \to \mathbb{R}^2$ linear transformation defined by $2 \times 2$ matrix $A$.

$$T(\vec{x}) = A\vec{x}.$$

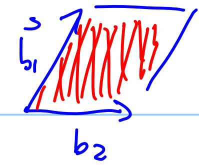


$S$ is parallelogram in $\mathbb{R}^2$.

$$\{\text{area of } T(S)\} = |\det A| \cdot \{\text{area of } S\}.$$

$$S = s_1\vec{b}_1 + s_2\vec{b}_2$$

$$0 \le s_1, s_2 \le 1$$




$$T(S) = T(s_1\vec{b}_1 + s_2\vec{b}_2)$$
$$= s_1 T(\vec{b}_1) + s_2 T(\vec{b}_2)$$

parallelogram defined by $T(\vec{b}_1)$ $T(\vec{b}_2)$

$$\left[ A\vec{b}_1 \quad A\vec{b}_2 \right] = A \cdot B$$

$$\underbrace{|\det(AB)|}_{\text{area } T(S)} = |\det A| \cdot \underbrace{|\det B|}_{\text{area}(S)}$$