Section 6.1

Note Title 11/20/2013

Inner Product $\vec{u}, \vec{v} \in \mathbb{R}^n$

Inner product of $\vec{u}$ + $\vec{v}$ $\quad \vec{u} \cdot \vec{v} = [u_1 \ldots u_n] \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$

$$= u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

$\vec{u} = (3, -1, 2)$

$\vec{v} = (1, 0, -1)$ $\quad \vec{u} \cdot \vec{v} = 3 \cdot 1 + -1 \cdot 0 + 2 \cdot (-1) = 1.$

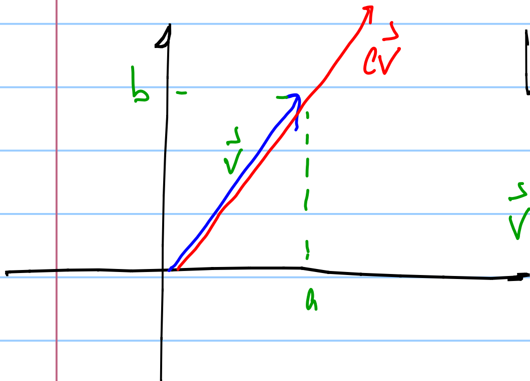
Theorem $\vec{u}\ \vec{v}\ \vec{w} \in \mathbb{R}^n$, c scaler.

1) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2) $(\vec{u} + \vec{v}) \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w}$
3) $c\vec{u} \cdot \vec{v} = \vec{v} \cdot c\vec{u}$
4) $\vec{u} \cdot \vec{u} \geq 0$ $\quad$ if $\vec{u} \cdot \vec{u} = 0$ then $\vec{u} = \vec{0}$

$\quad (\; u_1^2 + u_2^2 + \cdots u_n^2$

Length of $\vec{v} = \underline{\underline{\|\vec{v}\|}} = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$

$\|\vec{v}\|^2 = \vec{v} \cdot \vec{v}.$

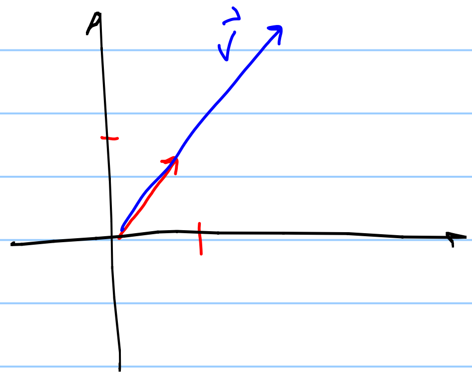


$\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ $\quad \|\vec{v}\| = \sqrt{a^2 + b^2}$

$$\|c\vec{v}\| = c\|\vec{v}\|$$

A vector of length 1 is called a unit vector.

Normalize a vector $\vec{v}$: $\frac{1}{\|\vec{v}\|}(\vec{v})$ ↳ unit vector in direction of v

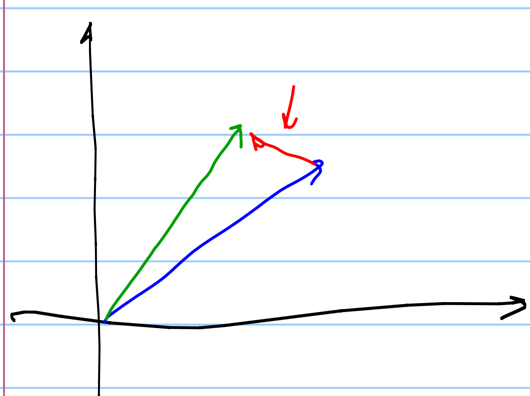


$$\vec{v} = (2, -1, -3, 1)$$

$$\|\vec{v}\| = \sqrt{2^2 + 1 + (-3)^2 + 1}$$

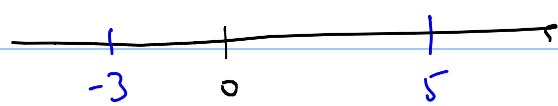
$$= \sqrt{15}$$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{15}}(2, -1, -3, 1) = \left(\frac{2}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}, \frac{1}{\sqrt{15}}\right)$$



$$\text{dist}(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|$$

In 1D



$$\text{dist}(a, b) = |a - b| \qquad \text{dist}(5, -3) = |5 - -3|$$

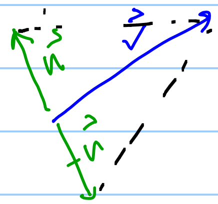
$$= |-3 - 5|$$

$\vec{u}, \vec{v} \in \mathbb{R}^n \qquad dist(\vec{u},\vec{v}) = \sqrt{(u_1-v_1)^2 + (u_2-v_2^2) + \cdots (v_n-v_n)^2}$

Orthogonality

Two vectors are orthogonal if

$$dist(\vec{v},\vec{u}) = dist(\vec{v},-\vec{u}).$$



$$dist(\vec{v},-\vec{u})^2 = \|\vec{v} - -\vec{u}\|^2 = \|\vec{v}+\vec{u}\|^2$$

$$= (\vec{v}+\vec{u})\cdot(\vec{v}+\vec{u})$$

$$= \vec{v}\cdot\vec{v} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{u}\cdot\vec{u}$$

$$= \|\vec{v}\|^2 + \|\vec{u}\|^2 + 2\vec{u}\vec{v}$$

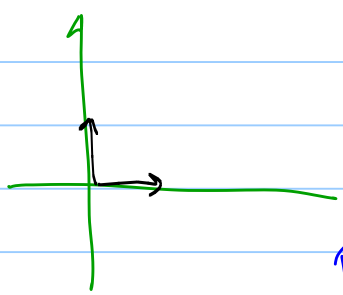
$$dist(\vec{v},\vec{u})^2 = \|\vec{v}-\vec{u}\|^2$$

$$= (\vec{v}-\vec{u})\cdot(\vec{v}-\vec{u})$$

$$= \|\vec{v}\|^2 + \|\vec{u}\|^2 - 2\vec{u}\vec{v}$$

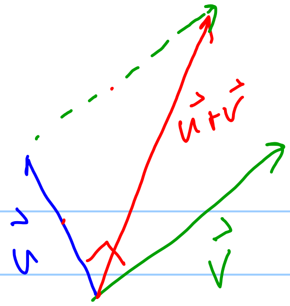
if $dist(\vec{v},\vec{u}) = dist(\vec{v},-\vec{u}) \Rightarrow 2\vec{u}\vec{v} = -2\vec{u}\vec{v} = 0$

$\vec{u}$ & $\vec{v}$ are orthogonal iff $\vec{u}\cdot\vec{v} = 0$.



$$\begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}0\\1\end{bmatrix} = 0.$$

Pythagorean Thm: $\vec{u}$ & $\vec{v}$ are orthogonal iff $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$



If W is a subspace of $\mathbb{R}^n$ (W is a plane in $\mathbb{R}^3$ through origin)

$W^\perp$ orthogonal complement of W

is all vectors that are orthogonal to all of W.

If $L = W^\perp$ $\quad L^\perp = W$.

A $m \times n$ matrix

$(\text{Row } A)^\perp = \text{Nul } A$

$(\text{Col } A)^\perp = \text{Nul } A^T$

$\text{Nul } A$: $\vec{x}$: $x \in \mathbb{R}^n$ $\quad A\vec{x} = \underline{\underline{\vec{0}}} \in \mathbb{R}^m$

$$\begin{array}{c} \vec{r}_1 \\ \\ \vec{r}_m \end{array}\begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \end{bmatrix}\begin{bmatrix} | \\ | \\ | \end{bmatrix} = \begin{bmatrix} \vec{r}_1 \cdot \vec{x} \\ \vec{r}_2 \cdot \vec{x} \\ \vdots \\ \vec{r}_n \cdot \vec{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A\vec{x} = \vec{0}$ iff $\vec{x}$ is orthogonal to every row of A

$\vec{x} \in (\text{Row } A)^\perp$