Note Title 12/4/2013

A eigenvector $\vec{v}$ $\qquad A\vec{v} = [\ \ ]$

$\lambda\vec{v}$

$A - I\lambda$

$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

$A = PDP^T = PDP^{-1}$

orthonormal.

Find eigenvalues:

$$\underbrace{\det \begin{bmatrix} 3-\lambda & -2 & 4 \\ -2 & 6-\lambda & 2 \\ 4 & 2 & 3-\lambda \end{bmatrix}}_{\text{deg 3 poly}} = 0$$

deg 3 poly $= 0$ solve for roots.

$(\lambda - 7)^2(\lambda + 2) = 0$ eigenvalues of A: 7 mult 2

-2 mult 1.

Find two (orthonormal) eigenvectors corresponding to $\lambda = 7$.

$$(A - 7I) = \underbrace{\begin{bmatrix} -4 & -2 & 4 \\ -2 & -1 & 2 \\ 4 & 2 & -4 \end{bmatrix}}_{B}$$

Solve $\underline{B\vec{x} = \vec{0}}$ (Sec 1.5)

$$B \rightsquigarrow \begin{bmatrix} 2 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$2x_1 + x_2 - 2x_3 = 0.$

Free vars: x_2 & x_3

$x_1 = -\frac{x_2}{2} + x_3$

$$\underline{\begin{bmatrix} -\frac{1}{2}x_2 + x_3 \\ x_2 \\ x_3 \end{bmatrix}} \to x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{a}_1 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} \quad \vec{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\vec{a}_1$ & $\vec{a}_2$ basis for eigenspace corresponding to eigenvalue $\lambda = 7$.

Gram - Schmidt.

$$\vec{v}_1 = \vec{a}_1 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 4/5 \\ 2/5 \\ 1 \end{bmatrix}$$

$$\vec{v}_2 = \vec{a}_2 - \frac{\vec{a}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \underline{\underline{\frac{+1/2}{1/4 + 1}}} \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2/5 \cdot -1/2 \\ 1 \cdot 2/5 \\ 0 \end{bmatrix}$$

$$\frac{-1/2}{5/4} = -\frac{2}{5}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/5 \\ 2/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4/5 \\ 2/5 \\ 1 \end{bmatrix}$$

$\lambda = -2$

$$(A - -2I) = \begin{bmatrix} 5 & -2 & 4 \\ -2 & 8 & 2 \\ 4 & 2 & 5 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -x_3$

$x_2 = -\frac{x_3}{2}.$

$$\vec{x}_3 = \begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix}$$

$$\vec{v}_3 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$$

Check $\vec{v}_1 \cdot \vec{v}_3 = 0$

$\vec{v}_2 \cdot \vec{v}_3 = 0.$

$$\vec{w}_3 = \begin{bmatrix} -2/3 \\ -1/3 \\ 2/3 \end{bmatrix}$$

$$\vec{v}_1 = \vec{a}_1 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 4/5 \\ 2/5 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$\frac{1}{\sqrt{v_2 \cdot v_2}} = \frac{1}{\sqrt{45}}$$

$$= \frac{1}{3\sqrt{5}}$$

$$\vec{w}_1 = \frac{1}{\sqrt{v_1 \cdot v_1}} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix}$$

$$\vec{w}_2 = \begin{bmatrix} \frac{4}{3\sqrt{5}} \\ \frac{2}{3\sqrt{5}} \\ \frac{5}{3\sqrt{5}} \end{bmatrix}$$

$$P = \begin{bmatrix} \vec{w}_1 & \vec{w}_2 & \vec{w}_3 \\ | & | & \end{bmatrix} \qquad D = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$