Note Title 10/8/2014

$P(x)$: $x+1 > x$

Domain: Set of real numbers.

$\forall x\, P(x)$

$Q(x)$: $x^2 > 0$

$\forall x\, Q(x) = \forall x\, x^2 > 0$. False

counterexample $x = 0$.

Existential Quantifier

$\exists x\, Q(x)$ "There exists an x...."

Domain $\{x_1, \ldots, x_n\}$

$Q(x_1) \vee Q(x_2) \vee \cdots \vee Q(x_n)$.

$Q(x)$: $x^2 > 0$ $\exists x\, Q(x)$?

Show the: example.

$\exists x \quad x+1 < x$

	True	False
$\forall x\, P(x)$	$P(x)$ true for all x	One counter example.
$\exists x\, P(x)$	One example.	$P(x)$ false for all x.

Domain: set of students at a high school.

$G(x)$: x went to the game.
$S(x)$: x is a senior.

Everyone is at the game: $\forall x\, G(x)$

Every senior went to the game:

$$\forall x\,(S(x) \to G(x))$$

There is a senior who did not go to the game.

$$\exists x\,(S(x) \wedge \neg G(x))$$

Set: collection of objects
elements.

$A = \{2, 4, 6, 8\} = \{8, 2, 4, 6\}$.

$2 \in A \qquad 5 \notin A$.

Empty set $\{\}$ ϕ

$\forall a \quad a \notin \phi$

Cardinality of a set is the # of elements.

$$|A| = 4$$

$D = \{1,2,3\}$ $\qquad$ $D = E$

$E = \{2,1,3\}$ $\qquad$ $x \in D$ if and only if $x \in E$.

$\mathbb{R}$ real number.

$\mathbb{N}$ natural number $\quad 1,2,3,\ldots$

$\mathbb{Z}$ integers. $\qquad \mathbb{Z}$ $\quad \mathbb{Z}^+$ positive integers.

$\mathbb{Q}$ rational number.

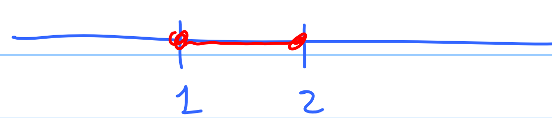
$A = \{2,4,6,8,\ldots,100\}$. $= \{x \in \mathbb{Z}^+ : x \text{ is even and } x \le 100\}$

$B = \{2,4,6,\ldots\}$

$A = \{x \in S : P(x)\}$

such that

$C = \{x \in \mathbb{R} : 1 \le x \le 2\}$



$A \subseteq B$. A is a subset of B.
if $x \in A$ then $x \in B$.

$A = \{x \in \mathbb{Z} : x \text{ is odd}\}$

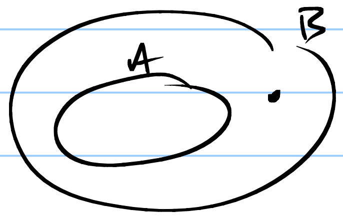
$B = \{3, 4, 5\}$.

$C = \{5, 7, 9\}$.

$B \overset{?}{\not\subseteq} A$

$C \subseteq A$ yes.

proper subset $A \subset B$ if $A \subseteq B$ and $A \neq B$



If $A \subset B$ then $A \subseteq B$.

Power set of A: set of all subsets of A.

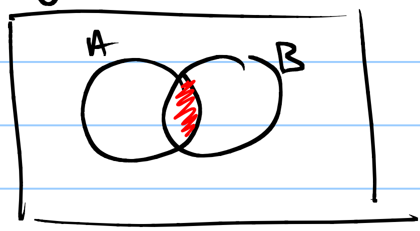
$A = \{a, b\}$ $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$\{a\} \in P(A)$

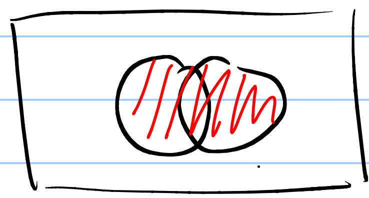
$\{a\} \subseteq A$.

$x \in A \cap B$ if $x \in A$ and $x \in B$

Universe Set



$A \cap B$.



$x \in A \cup B$ if $x \in A$ or $x \in B$.

$A = \{x \in \mathbb{Z} \mid x \text{ is prime}\}$

$B = \{x \in \mathbb{Z} \mid x \text{ is even}\}$

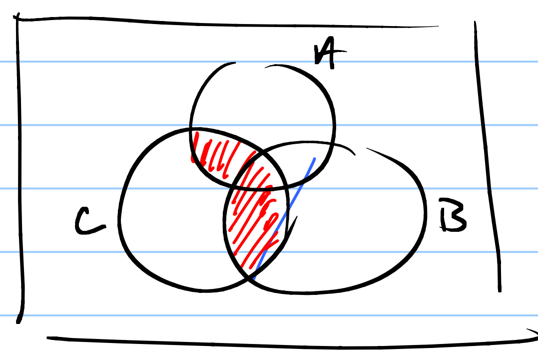
$C = \{x \in \mathbb{Z} \mid 1 < x < 20\}$.

$D =$

$A \cap B = \{2\}$.

$B \cap C = \{2, 4, \ldots, 18\}$.

$(A \cup B) \cap C$



$A = \{1, 2, 4, 7, 8\}$

$B = \{1, 3, 7, 9, 10, 11\}$.

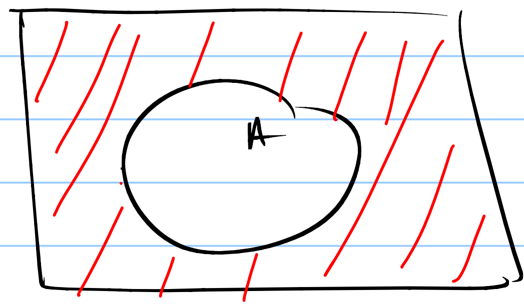
$C = \{x \in \mathbb{Z} : x \text{ prime}\}$.

$(B \cap C) \cup A$.

$\{3, 7, 11\}$

$\{1, 2, 3, 4, 7, 8, 11\}$.

Complement: define Universe Set U:



$\bar{A}$

$U = \{1, 2, 3, \ldots, 12\}$

$A = \{1, 2, 4, 7, 8\}$

$\bar{A} = \{3, 5, 6, 9, 10, 11, 12\}$

$B = \{1, 3, 7, 9, 10, 11\}$.

$\bar{A} \cup B = \{1, 3, 5, 6, 7, 9, 10, 11, 12\}$

$\overline{A \cup B} = \{5, 12, 6\}$

$A \cup B = \{1, 2, 3, 4, 7, 8, 9, 10, 11\}$

Cartesian product : $A \times B$.

set of all ordered pairs.

$A = \{1,2,3\}$

$B = \{a,b\}$

$(\_\_ , \_\_)$

in A in B.

$A \times B = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$.

$(2,a) \in A \times B$.

$(a,b) \neq (b,a)$.

parens.

$B \times B = \{(a,a),(a,b),(b,a),(b,b)\}$

$B \times B = B^2$

$\mathbb{R}^2 \sim$ 2d plane.

Drinks = {coffee, OJ}.

Main = {french toast, pancakes, eggs}.

Side = {hash browns, bacon}.

Drink × Main × Side $\ni$

(OJ, french toast, bacon) $\in$

$\mathbb{R}^3$

$B = \{0, 1\}$ B^7

$\{0,1\}^3 = \{000, 001, \ldots, 111\}$

Passwords 6, 7, 8 chars in length
lower case letters

$L = \{a, b, \ldots, z\}$ $L^6 \cup L^7 \cup L^8$

$\{+, *\} \times L^6$

$\{0,1\}^n$