Fri, Oct 10.

Note Title 10/10/2014

$f(x) = x^2 - 3$

$g(x) = \sin^2(x)$

$f : A \to B$

A, B sets

inputs

A domain

B target (co-domain).

outputs.

$f : \{0, 1\}^5 \to \{0, 1\}^4$

$f(01101) = 0101$

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = 1/x \quad x = 0.$

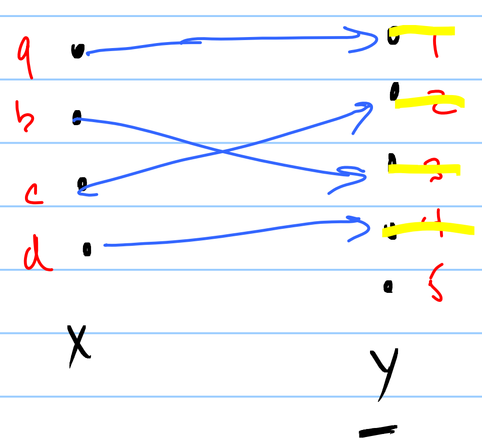
$f(x) = \pm\sqrt{x^2+1}$

not well defined.

$g : X \to Y$

$X = \{a, b, c, d\}$

$Y = \{1, 2, 3, 4, 5\}$



$f(a) = 1$

$f(b) = 3$

$f(c) = 2$

$f(d) = 4$

The range of a function $\subseteq$ Target

$f: X \to Y$

Range of f = $\{y \in Y : \exists\, x \in X \text{ and } f(x) = y\}$.

$g: \mathbb{R} \to \mathbb{R} \qquad g(x) = x^2$

$h: \mathbb{Z} \to \mathbb{Z} \qquad h(x) = 3x$

$f: \{0,1\}^2 \to \{0,1\}^3$ $\qquad$ $f(x)$ = copy the first bit and append to end.

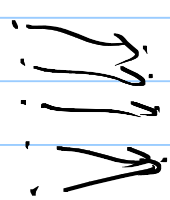
$$\left.\begin{array}{l} f(10) = 101 \\ f(01) = 010 \\ f(11) = 111 \\ f(00) = 000 \end{array}\right\}.$$

~~001~~

A function is <u>onto</u> if Range = Target

if $\forall\, y \in T\ \exists\, x \in X\ f(x) = y$.

$f: X \to Y$ and f is onto. $\Rightarrow |X| \geq |Y|$.



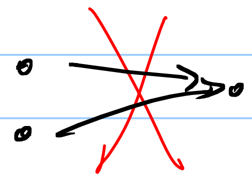
$c: \{0,1\}^3 \to \{0,1\}^3$

$c(x)$: Remove first bit and append to end.

$c(100) = 001$ onto.
$c(011) = 110$.

A function $f: X \to Y$ is one-to-one

if $x, x' \in X$ $x \neq x'$ then $f(x) \neq f(x')$



If f is 1-1 then $|Y| \geq |X|$

$g: \mathbb{Z} \to \mathbb{Z}$ $g(x) = x^2$ 1-1? $g(2) = g(-2) = 4$

$c(x)$ cyclic shift left. 1-1.

$f: \{0,1\}^2 \to \{0,1\}^2$ $f(\underset{b_1 b_2}{x}) = b_1 \cdot b_2\, 0$.

$f(11) = 10$
$f(01) = 00$
$f(10) = 00$ not 1-1.

$g: \mathbb{Z} \to \mathbb{Z}$ $g(x) = x+2$ 1-1, onto.

$g: \mathbb{N} \to \mathbb{N}$ $g(x) = x+2$ 1-1, Not onto.

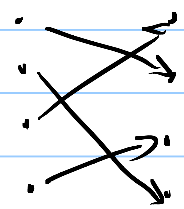
If $g: X \to Y$ is 1-1 and onto.

it is a <u>bijection</u>

$\Updownarrow$

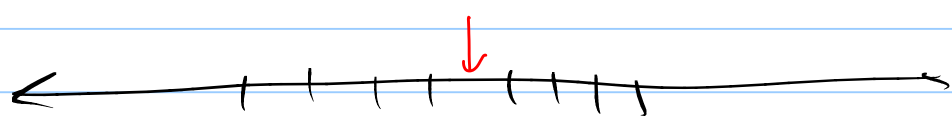
"1-1 correspondence".

then $|X| = |Y|$



floor: $\mathbb{R} \to \mathbb{Z}$. $\lfloor x \rfloor$ = floor(x).

largest integer $\le x$.



$\lfloor 4.9 \rfloor = 4$ $\lfloor -4.1 \rfloor = -5$

$\lfloor 4 \rfloor = 4$

ceiling: $\mathbb{R} \to \mathbb{Z}$ $\lceil x \rceil$ = smallest int $\ge x$.

$\lceil -5.9 \rceil = -5$.

Sequence $a_0, a_1, a_2, \ldots$ $\{a_i\}$ sequence.

a_i element at index i

$c_1, c_2, \ldots$

$d_{-2}, d_{-1}, d_0, \ldots$ $\qquad$ d_1, d_2, d_3, d_4.

$f_0, f_1, f_2, \ldots$ $\qquad$ $f_n = 2^n + n$.

$f_0 = 1$
$f_1 = 1$ $\qquad$ for $n \geq 2$. $\qquad$ $f_n = f_{n-1} + f_{n-2}$.

Geometric Sequence : $\qquad$ $c_1 = d$.

common ratio r

$c_n = r \cdot c_{n-1}$

Increasing: if $r > 1$
Decreasing if $r < 1$.

Arithmetic $\qquad$ $b_0 = a$.

$b_n = b_{n-1} + d$.

Summation: $\qquad$ $1^2 + 2^2 + 3^2 + \cdots + 17^2$

$$\sum_{j=1}^{17} j^2$$

17 — upper limit

$j=1$ — lower limit.

$$\sum_{j=0}^{n} a_j = a_0 + a_1 + \cdots + a_n$$

$$= \sum_{k=0}^{n} a_k.$$

The sum of all non-negative multiples of 3 less than 300.

$$\sum_{k=0}^{99} 3\cdot k = \sum_{k=1}^{99} 3k$$

$$\sum_{k=0}^{99} (3k+1)$$

~~Geometric:~~ Arithmetic $b_0 = a.$ $b_n = d + b_{n-1}$ $b_n = a + nd.$

$a, a+d, a+2d, \ldots$

$\uparrow$ 0 $\uparrow$ 1

$$\sum_{j=0}^{n} b_j = \sum_{j=0}^{n} (a+jd)$$

$$= a(n+1) + \frac{d\,n(n+1)}{2}.$$

$$\sum_{j=0}^{n-1} (a+jd) = an + \frac{d(n-1)n}{2}.$$

$$\sum_{j=1}^{n} j = 1 + 2 + 3 + \cdots + n = S$$

$$\sum_{j=1}^{n} j = n + (n-1) + (n-2) \cdots 1 = S$$

$(n+1) + (n+1) \cdots (n+1) = 2S$

$$2S = (n+1)n. \qquad S = \frac{n(n+1)}{2} = \sum_{j=1}^{n} j.$$

$$\sum_{j=1}^{33} 3j = 3 + \sum_{j=2}^{33} 3j = \left[\underline{\sum_{j=1}^{32} 3\cdot j}\right] + \underline{99}.$$