

$\{0, 1\}^7$  set of binary strings of length 7.

$(-7 * 8^2)$   $(-7 * 8^2)$   
 $-7 * 8^2$   $(-7 * 8^2)$

$((())())$

$B = \{0, 1\}$

$B^{17}$  binary strings  
of length 17.

$B^*$  the set of all finite length  
binary strings.

$\lambda$  = empty string.

length of  $\lambda$  is 0 ""

$B^* = B^0 \cup B^1 \cup B^2 \cup B^3 \cup \dots$   
 $\{ \lambda \}$

Recursive def of  $B^*$

concat

1) Base case:  $\lambda \in B^*$   
 2) If  $x \in B^*$   
 then  $x0 \in B^*$   $x1 \in B^*$

101.

$\lambda \rightarrow \lambda 1 = 1 \rightarrow \underline{10} \rightarrow 101$   
 $\in B^* \quad \in B^* \quad \in B^*$

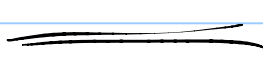
Length of string  $x$   $|x|$  # chars,

$$|1101| = 4.$$

Length of a string in  $B^*$  1)  $| \lambda | = 0$ . Base case

2)  $x \in B^*$   $|x0| = |x| + 1$   
↳ Recursive rule.  $|x1| = |x| + 1$ .

$\lambda \rightarrow 1 \rightarrow 10 \rightarrow 101$   
0      1      2      3.



$(())$        $(())()$        $()(())$

Set of properly nested strings of  $()$

Base:  $()$

Recursive rule: A) If  $x$  is properly nested  
then  $(x)$  prop nested.

B) If  $x + y$  are properly nested  
then  $xy$  is properly nested.

$((())())$

→ 1)  $()$  base case.

→ 2)  $(())$  RR A.

3)  $(())()$  RR B.

4)  $((())())$

$$\underline{n! = n(n-1)(n-2) \dots 2 \cdot 1}$$

Base case:  $0! = 1$ .

Inductive Rule:  $n! = n \cdot (n-1)!$

Re

$$1! = 1 \cdot 0! = 1 \cdot 1 = 1.$$

$n=1$

Recursively defined sequences: initial conditions (base case)  
recurrence relation (recursively)

Fibonacci:  $f_0 = 1$   
 $f_1 = 1$  } initial conditions.

$$\text{For } n \geq 2 \quad f_n = \underline{\underline{f_{n-1} + f_{n-2}}}$$

1, 1, 2, 3, 5, 8, ...

$$g_0 = 2 \quad g_n = 3 \cdot g_{n-1} + n \quad \leftarrow n \geq 1$$

$$g_0 = 2$$
$$g_1 = 3 \cdot g_0 + 1 = 3 \cdot 2 + 1 = 7$$

$$g_2 = 3 \cdot g_1 + 2 = 3 \cdot 7 + 2 = 23$$

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$$t_n = 3 \cdot t_{n-1} + 2 \cdot t_{n-3}$$

$$t_1 = 1$$

$$t_2 = 2$$

$$t_3 = 1$$

$$t_4 = 3 \cdot t_3 + 2 \cdot t_1$$
$$= 3 \cdot 1 + 2 \cdot 1 = 5$$

$$t_n =$$

Factorial (n) non-neg int.  $0! = 1$ .

If (n=0) Return 1.

Return (n \* Factorial (n-1))

End

Power (a, n) //  $a^n$  a real #  
n non-neg integer.

If (n=0) Return 1.

Return (a \* Power (a, n-1))

End;

Power (a, n)

$$\underline{a^{18}} = \left( \underline{a^9} \right)^2$$

If (n=0) Return 1.

If n is even

Return (Power (a, n/2)<sup>2</sup>);

If n is odd

Return (a \* (Power (a,  $\lfloor n/2 \rfloor$ ))<sup>2</sup>);

$$\left( a^{n/2} \right)^2 =$$

$$a^{n/2 \cdot 2} = a^n$$

$$a^{2k+1} = a^{2k} \cdot a$$

End;

$$a^{19} \quad \lfloor 19/2 \rfloor = 9.$$