Note Title 10/15/2014

Induction:

$P(n)$: predicate, domain is non-neg integers.

Theorem: For any integer $n \geq c$. $P(n)$ is true.

↳ const. 1, 2, 3, ...

It is sufficient to prove: ⇒ (1) $P(c)$ is true. Base case.

Inductive step. ⇒ (2) If $n \geq c$ and $P(n)$ is true then $P(n+1)$ is also true

$P(c) \to P(c+1)$ ✓

$P(c+1) \to P(c+2)$ ✓

$P(c+2) \to P(c+3)$

$$P(c) \to P(c+1) \to P(c+2) \to \ldots$$

* Theorem: For every $n \geq 1$, $\underbrace{\sum_{j=1}^{n} j = \frac{n(n+1)}{2}}_{P(n)}$.

$P(3)$: $1+2+3 = \frac{3(3+1)}{2}$.

$P(17)$: $\sum_{j=1}^{17} j = \frac{17(17+1)}{2}$

$P(k+1) = \sum_{j=1}^{k+1} j = \frac{(k+1)(k+2)}{2}$.

Proof Base case! $n=1$ Prove $\sum_{j=1}^{1} j = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$\sum_{j=1}^{1} j = 1$ ✓

Inductive Step! Assume $n \geq 1$ $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$

Prove $\sum_{j=1}^{n+1} j = \frac{(n+1)(n+2)}{2}$.

$$\sum_{j=1}^{n+1} j = \_\_ = \_\_ = \_\_ = \_\_ = \frac{(n+1)(n+2)}{2}.$$

$$\sum_{j=1}^{n+1} j = (n+1) + \sum_{j=1}^{n} j = (n+1) + \frac{n(n+1)}{2}.$$ by the inductive hypothesis.

$$= \frac{2(n+1) + n(n+1)}{2}.$$

$$= \frac{(n+1)(2+n)}{2}. \quad \boxtimes$$

Thm for $n \geq 1$, $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.

Proof Base case $n=1$! $\sum_{j=1}^{1} j^2 = \frac{1\cdot(1+1)(2\cdot 1+1)}{6}$

$1^2 = 1$ $\qquad \frac{6}{6} = 1$.

$$\sum_{j=1}^{7} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

Inductive Step: Assume $\displaystyle\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$

Prove: $\displaystyle\sum_{j=1}^{n+1} j^2 = \frac{(n+1)(n+2)(2\cdot(n+1)+1)}{6}$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\sum_{j=1}^{n+1} j^2 = (n+1)^2 + \sum_{j=1}^{n} j^2 = (n+1)^2 + \frac{n(n+1)(2n+1)}{6}$$ By the I.H.

$$= \frac{6(n+1)^2 + n(n+1)(2n+1)}{6}.$$

$$= \frac{(n+1)\left[6(n+1) + n(2n+1)\right]}{6}$$

$$= \frac{(n+1)\left[6n+6 + 2n^2+n\right]}{6}$$

$$= \frac{(n+1)\left[2n^2+7n+6\right]}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$ ☒

$2^{2n}-1 = 3k$ some int k.

Theorem: For every $n \geq 1$ 3 evenly divides $2^{2n}-1$.

$Q(n)$.

$Q(2)$: 3 evenly divides $2^{2\cdot 2}-1 = 2^4-1 = 15$

$Q(3)$: 3 evenly divides $2^{2\cdot 3}-1 = 2^6-1 = 63$