Note Title 10/20/2014

Define sequence $\{g_n\}$

$$g_0 = 1$$
$$g_n = \boxed{3 \cdot g_{n-1} + 2n} \quad *$$

$g_0 = 1, \quad g_1 = 3 \cdot 1 + 2 \cdot 1 = 5, \quad g_2 = 3 \cdot 5 + 2 \cdot 2 = 19, \ldots$

Theorem: for $n \geq 0$ $\quad g_n = \frac{5}{2} \cdot 3^n - n - \frac{3}{2}$.

Base: $n = 0$. $\quad \frac{5}{2} \cdot 3^0 - 0 - \frac{3}{2} = \frac{5}{2} - \frac{3}{2} = 1$ ✓

$g_0 = 1$ ✓

Ind Step: Assume $n - 1 \geq 0$ $\quad g_{n-1} = \frac{5}{2} \cdot 3^{n-1} - (n-1) - \frac{3}{2}$

Prove $\quad g_n = \frac{5}{2} \cdot 3^n - n - \frac{3}{2}$.

$$\begin{aligned}
g_n &= 3 \cdot g_{n-1} + 2n \quad \text{(by def)} \\
&= 3\left(\frac{5}{2} \cdot 3^{n-1} - (n-1) - \frac{3}{2}\right) + 2n \quad \text{(by I.H.)} \\
&= \frac{5}{2} \cdot 3 \cdot 3^{n-1} - 3(n-1) - 3 \cdot \frac{3}{2} + 2n \\
&= \frac{5}{2} \cdot 3^n - 3n + 3 - \frac{3 \cdot 3}{2} + 2n \\
&= \frac{5}{2} \cdot 3^n - n - \frac{3}{2} \quad \boxtimes
\end{aligned}$$

**Theorem:** For every $n \geq 1$ 3 evenly divides $2^{2n}-1$

[$2^{2n}-1 = 3k$ for some int $k$].

Pf Base case: $n=1$ $\quad 2^{2\cdot 1}-1 = 4-1 = 3$

3 evenly divides 3.

Induction Step: Assume $n \geq 2$ 3 evenly divides $2^{2(n-1)}-1$.

Prove 3 evenly divides $2^{2n}-1$.

Induction hypothesis: $\boxed{2^{2(n-1)}-1 = 3k}$ for some int $k$.

$$
\begin{aligned}
2^{2n}-1 &= 2^2 \cdot 2^{2n-2} - 1 \\
&= 4 \cdot 2^{2(n-1)} - 1 \\
&= 4 \cdot 2^{2(n-1)} - 4 + 3 \\
&= 4\,(2^{2(n-1)}-1) + 3 \\
&= 4\,(3k) + 3. \qquad \text{(I.H.)} \\
&= 3\,\underbrace{(4k+1)}_{\text{int}}
\end{aligned}
$$

int so 3 divides $2^{2n}-1$.

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**Strong induction:**

Base cases: $P(a) \wedge P(a+1) \wedge \cdots P(b)$.

Assume for $k = a, \ldots, n-1$ $P(k)$ is true.

$n-1 \geq b$.

$P(a) \wedge P(a+1) \wedge \cdots \wedge P(b) \wedge \cdots P(n-1)$.

Prove $P(n)$ is true.

**Weak.**

Base: $P(a)$ is true

Assume $P(n-1)$ is true

$n-1 \geq a$

Prove $P(n)$ is true.

Theorem: Every amount of postage at least 12¢ can be formed w/ 4¢ & 5¢ stamps.

Base:

$$\begin{cases} 12¢ & - \quad 3 \times 4¢ \\ 13¢ & - \quad 2 \times 4¢ + 5¢ \\ 14¢ & \quad 1 \times 4¢ + 2 \times 5¢ \\ 15¢ & \quad 3 \times 5¢ \end{cases}$$

Assume: For $k = 12, 13, \ldots, \underline{n-1}$ $(n-1 \geq 15)$ can make $k$¢.

Prove: Can $n$¢

$n-1 \geq 15$

$\underline{n-4 \geq 12}$

By I.H can make $(n-4)$¢

add one 4¢ stamp

$\Rightarrow$ $n$¢!