Note Title 11/1/2014

Product Rule: Finite Sets $A_1, \dots, A_n$

$(a_1, a_2, \dots, a_n)$

$$|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$$

Lunch Special at a restaurant, Selection of Sandwich, side and a drink.

Sandwiches = {Burger, Grilled chicken, Grilled cheese}.
Sides = {Fries, Fruit}.
Drinks = {Coke, Diet Coke, OJ, 7UP}

A meal selection specified by a triple:

([Sandwich], [Side], [Drink])

For example: (Burger, fries, coke).

Total # of distinct choices for lunch specials:

$$|\text{Sandwiches}| \times |\text{Sides}| \times |\text{Drinks}| =$$

$$3 \times 2 \times 4 = 24.$$

Process of making a selection:
Multiply # selections at each point.

* [Product rule works when the # choices at each point is independent of the choices made so far.] *

A school has 5 4^{th} grade classes.
The student council consists of one representative chosen from each class.

C_i = set of kids in the i^{th} class. For $i = 1, 2, 3, 4, 5$

ways to select student council:

$$|C_1| \cdot |C_2| \cdot |C_3| \cdot |C_4| \cdot |C_5|$$

Athlete training for a triathalon makes exercise schedule for the week. For each of the 7 days, she can run, swim or bike.
How many different schedules are possible?

S	R	R	B	B	S	S
M	T	W	Th	F	Sa	Su.

$$3 \cdot 3 \cdot 3 \cdot \ldots \cdot 3 = 3^7$$

Counting strings of length n.

Set of symbols = Σ.

How many strings of length $\underline{n}$ w/ symbol set $\underline{\Sigma}$?

$$|\Sigma^n| = |\Sigma|^n$$

binary strings of length 15 $= 2^{15}$

Specify each string ~~as a~~ selection process:
(binary strings of length 3).

$x \in \{0,1\}^4$

$\underline{0/1}\ \underline{0/1}\ \underline{0/1}\ \underline{0/1}$

$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

Sum Rule: If $A_1, \ldots, A_n$ are pairwise disjoint (if $i \neq j \Rightarrow A_i \cap A_j = \phi$) then
$$|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|.$$

Sandwiches = {Burger, Grilled chicken, Grilled cheese}.
Sides = {Fries, Fruit}.
HotDrinks = {coffee, tea, cocoa}.
ColdDrinks = {water, soda}.

Lunch consists of a Sandwich
a Side
& a Drink $\leadsto$ $\text{HotDrinks} \cap \text{ColdDrinks} = \phi$

$\text{Drinks} = \text{HotDrinks} \cup \text{ColdDrinks}$
$|\text{Drinks}| = |\text{HotDrinks}| + |\text{ColdDrinks}|$
$= 2+3 = 5$

Lunch Selections =

$$3 \cdot 2 \cdot (2+3) = 3 \cdot 2 \cdot 5 = 30.$$

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$L = \{A, B, C, D, \ldots, Y, Z\}.$

System passwords: String of digits or letters of length 7.

passwords: $\underset{\substack{36 \\ 4}}{\underline{36}} \cdot \underline{36} \cdot \underline{36} \; \underline{\quad} \; \underline{\quad} \; \underline{\quad} \; \underline{36} = (36)^7$

- length 7, must start w/ a letter:

$$26 \cdot (36)^6$$

- length 6, 7, or 8.

$$(36)^6 + (36)^7 + (36)^8$$

Bijection Rule:

If A & B are finite sets and there is a bijection between A & B. ($f: A \to B$ where f is a bijection)
then $|A| = |B|$.

1 to 1
onto.

Let S be a finite set and $|S| = n$.

$$S = \{x_1, x_2, \dots, x_n\}.$$

$$|P(S)| = 2^n$$

Power set of S: set of all subsets.

$$f: P(S) \to \{0,1\}^n.$$

$A \subseteq S$ $\quad f(A) = n$-bit string.

j^{th} bit is 0 if $x_j \notin S$
j^{th} bit is 1 if $x_j \in S$.

$$S = \{x_1, x_2, x_3, x_4, x_5\}.$$

$A = \{x_1, x_5\} \Rightarrow f(A) = 10001$

$A' = \{x_2, x_3, x_5\}$ $\quad f(A') = 01101$

Why is f a bijection?
Each subset of S uniquely specifies an n-bit string.
Each n-bit string uniquely specifies a subset of S.

10101 $\quad A = \{x_1, x_3, x_5\}$

$f(A) = 10101$.

E = Set of 5-bit strings w/ an even # of 1's.

$10110 \notin E$

$10111 \in E$.

$00000 \in E$.

Bijection between $f: \{0,1\}^4 \to E$.