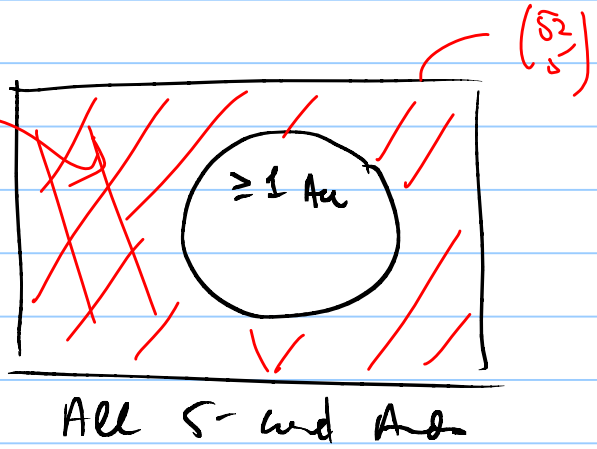
Board Notes 11-10-14

Note Title 11/9/2014

Back to 5-card hands from a standard playing deck.

- # hands w/ at least one Ace:

hands w/ no aces: $\binom{48}{8}$



$$\binom{52}{8} - \binom{48}{5}$$

$\binom{52}{8}$: all hands; $\binom{48}{5}$: hands w/ no aces.

hands with exactly one King or exactly one Ace (or both)

$|K|$ = # hands w/ 1 king. $= 4 \cdot \binom{48}{4}$

$|A|$ = # hands w/ 1 ace $= 4 \cdot \binom{48}{4}$

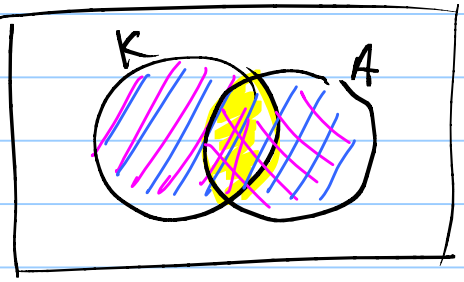
Over counting $|K \cap A| = 4 \cdot 4 \cdot \binom{44}{3}$

$4 \cdot \binom{48}{4} + 4\binom{48}{4} - 4 \cdot 4 \cdot \binom{44}{3}$

A = set of hands w/ exactly one Ace.

K = set of hands w/ exactly one King.

$|A \cup K| = |A| + |K| - |A \cap K|$



Inclusion-Exclusion Principle w/ 2 Sets:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

A: set with constraint a.
B: set with constraint b.

Set w/ constraint a or b is $A \cup B$.

Example: Strings of length 10 over {a, b, c}

Total #: 3^{10}

Start with a 3^9 a ⟶ 9

Start or end with a.

Start w/ a: 3^9
End w/ a: 3^9
Start + End w/ a: a ⟶ a. 3^8

$3^9 + 3^9 - 3^8$.

Start with a or b. $3^9 + 3^9$

a ⟶ 9 ← 3^9
b ⟶ 9 ← 3^9

No intersection!

at least one a:

$$3^{10} - 2^{10}$$

3^{10} ↳ total # strings

2^{10} → string w/ no a's.

contains 9 consecutive a's:

aaaaaaaaa* OR *aaaaaaaaa

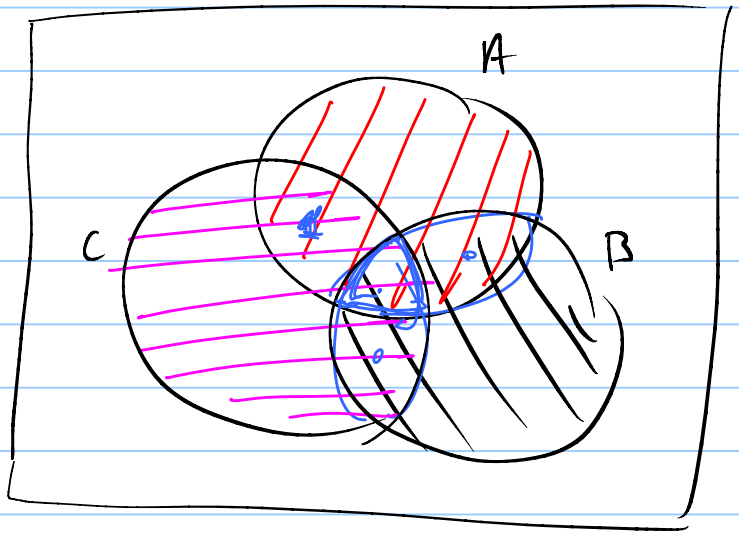
↓

$$3 + 3 - 1 = 5$$

aaaaaaaaaaa could trip

$|A \cup B \cup C|$

$|A| + |B| + |C| \Leftarrow$
$- |A \cap B| - |B \cap C|$
$- |A \cap C|$
$+ |A \cap B \cap C|$



Inclusion/Exclusion w/ 3 sets: (animation).

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|.$$

Drug test on a population of 1000.

- 122 people develop symptom A.
- 88 people develop symptom B.
- 112 people develop symptom C.

- 27 get A & B
- 29 get B + C
- 32 get A + C.

- 10 get all 3 (A, B + C).

$$122 + 88 + 112 - 27 - 29 - 32 + 10 = \boxed{}$$

How many get at least one symptom?