Nov 12, 2014

Note Title 11/12/2014

Line up of 7 people.

Mother, Father, 3 sons, 2 daughters.

How many is mother next to at least one of her 3 sons?

A, B, C

S_A set of line ups w/ mom next son A.
S_B B
S_C C

$$|S_A \cup S_B \cup S_C| = |S_A| + |S_B| + |S_C|$$
$$- |S_A \cap S_B| - |S_B \cap S_C| - |S_A \cap S_C|$$
$$+ \cancel{|S_A \cap S_B \cap S_C|}\ 0$$
$$= 3(2 \cdot 6! - 2 \cdot 5!).$$

$$|S_A| = \underset{\text{how to left or right of A?}}{2} \cdot 6! = |S_B| = |S_C|.$$

D, M-A, B, C, 1, 2

$$|S_A \cap S_B| = 2 \cdot 5!$$

D, B-M-A, C, 1, 2.

Back to strings of length 10 over $\{a,b,c\}$.

How many have 8 consecutive a's?

S $\quad aaaaaaaa**$ $\longrightarrow 9$

R $\quad *aaaaaaaa*$ $\longrightarrow 9$

T $\quad **aaaaaaaa$ $\longrightarrow 9$

$$\overset{9}{|S|} + \overset{9}{|R|} + \overset{9}{|T|} - \overset{3}{|S \cap R|} - \overset{1}{|S \cap T|} - \overset{3}{|R \cap T|} + \overset{1}{|S \cap R \cap T|}$$

$S \cap R$: $aaaaaaaaa*$

$S \cap T$ $\quad a$ ———— a $= S \cap R \cap T$

$R \cap T$ $\quad *aa - - - - a$

$$9 + 9 + 9 - 3 - \cancel{1} - 3 + \cancel{1} = 21.$$

$aaaaaaaa\cancel{*}^{a}*$

$\cancel{*}_{a}aaaaaaaa*$

Binomial Coefficients and Combinatorial Arguments.

Multiplying polynomials:

$$(x+y)(x+y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = (x+y)^2(x+y)$$

$$\begin{aligned} x(x+y)^2 &= x^3 + 2x^2y + y^2x \\ + \ y(x+y)^2 &= \phantom{x^3 + {}} x^2y + 2y^2x + y^3 \\ \hline & x^3 + 3x^2y + 3y^2x + y^3 \end{aligned}$$

Multiply out $(x+y)^3 = xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$

$$\underset{x \text{ or } y}{(x+y)}\ \underset{x \text{ or } y}{(x+y)}\ \underset{x \text{ or } y}{(x+y)}$$

To get coefficient of x^2y term: 3

$$\left.\begin{matrix} x\,x\,y \\ x\,y\,x \\ y\,x\,x \end{matrix}\right\} 3$$

# of strings of length n w/ k x's, n-k y's. $\binom{n}{k}$

$$\underset{1}{\_\_}\ \underset{2}{\_\_}\ \underset{3}{\_\_}\ \underset{4}{\_\_}\ \_\_\ \_\_ \cdots \underset{n}{\_\_}$$

Generalize: $(x+y)^n \Longrightarrow$ Sum over all "strings" of length n over $\{x,y\}$.

Coefficient of $x^k y^{n-k}$ = # of strings w/ k x's and n-k y's $= \binom{n}{k}$

Binomial Theorem: For any x, y.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} \underline{x^k y^{n-k}}.$$

Expand $(x+y)^5 = \binom{5}{5} x^5 + \binom{5}{4} x^4 y + \binom{5}{3} x^3 y^2$

$+ \binom{5}{2} x^2 y^3 + \binom{5}{1} x y^4 + \binom{5}{0} y^5$

Apply to: $(3a - 2b)^6$ What is coeff $\underline{a^4 b^2}$

$(\underbrace{3a}_{x} + \underbrace{(-2b)}_{y})^6$

$\binom{6}{4} x^4 y^2 = = \binom{6}{4} (3a)^4 (-2b)^2$

$= \binom{6}{4} 3^4 (-2)^2 a^4 b^2.$

$(\underbrace{-4a}_{x} + \underbrace{b}_{y})^9$ coeff $a^3 b^6$

$\binom{9}{3} x^3 \cdot y^6 = \binom{9}{3} (-4a)^3 (b)^6 = \underbrace{\binom{9}{3} (-4)^3}\, a^3 b^6$

$- \binom{9}{3} 4^3 a^3 b^6$

Can use Binomial Theorem to prove identities.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Plug in $x=y=1$.

$$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k}$$

$$2^n$$

$$\boxed{2^n = \sum_{k=0}^{n} \binom{n}{k}}$$

Combinatorial Argument.

$$\#\text{ Subsets of } \{1,\dots,n\} = \sum_{k=0}^{n} \left(\#\text{ Subsets of } \{1,2,\dots,n\} \text{ of size } k\right)$$

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Pascal's Identity: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

k-subsets of $\{\cancel{1}, \underbrace{2, \ldots, n, n+1}\}$ = # k-subsets of $\{1, 2, \ldots, n, n+1\}$ that do not include 1 + # k-subsets of $\{1, 2, \ldots, n, n+1\}$ that DO include 1.

$\{x_1, x_2, \ldots, x_n\}$

All 3-subsets of $\{1, 2, 3, 4, 5\}$

$n+1 = 5$

$k = 3$.

Have 1

$\{1,2,3\}$ $\{1,3,4\}$
$\{1,2,4\}$ $\{1,3,5\}$
$\{1,2,5\}$ $\{1,4,5\}$

$\{2,3,4\}$ $\{2,3,5\}$
$\{2,4,5\}$

Don't Have 1.