Pigeonhole Principle.

Note Title 11/12/2014

pigeons → pigeonholes

$f: A \to B$ and $|A| > |B|$ then f is not one-to-one.

$\exists \; , a' \in A \quad f(a) = f(a')$.

Among any 13 people there must be at least 2 who are born in the same month.

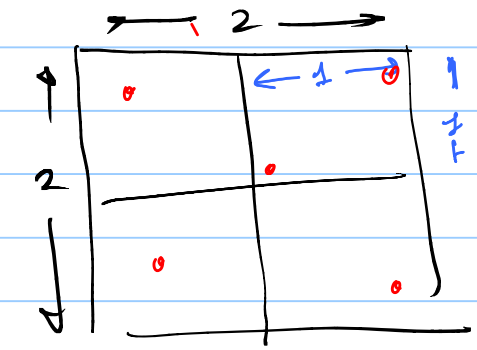
f: person → birthday month.

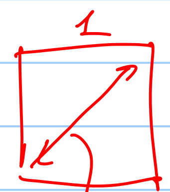
12.

$|\text{persons}| \geq 13$

Chain is planning to put five stores in an area that is 2 miles × 2 miles.

⇒ there will be two stores that are within $\sqrt{2}$ miles from each other.





$\sqrt{1^2 + 1^2} = \sqrt{2}$.

3 kids on a track team compete in the high jump.

All 3 have PR's ≥ 6 ft < 7 ft

Will there be two whose PR is within 4" of each other.

6' 1"
6' 5.5"
6' 10"

What about 4 kids?

3 intervals: [6', 6', 4") [6' 4", 6' 8") [6' 8", 7')

Set S of 26 6-digit #'s:

There will be two different subsets of the 26 #'s whose elements sum to the same value.

$x_1 = 562137$
$x_2 = 112149$
$x_3 = 914326$
$x_4 = 511224$
$x_5 = 818479$
$x_6 = 922567$
$x_7 = 425631$
$x_8 = 122496$
$x_9 = 153427$
$x_{10} = 751631$
$x_{11} = 721456$
$x_{12} = 131572$
$x_{13} = 721879$
$x_{14} = 872351$
$x_{15} = 723156$
$x_{16} = 562131$
$x_{17} = 621052$
$x_{18} = 732010$
$x_{19} = 400562$
$x_{20} = 314900$
$x_{21} = 492671$
$x_{22} = 279421$
$x_{23} = 261731$
$x_{24} = 885612$
$x_{25} = 134672$
$x_{26} = 625413$

Example $\{\underline{x_7}, \underline{x_{21}}, \underline{x_{22}}\} \rightarrow$

$425631 + 492671 + 279421$

distinct subsets of the 26 numbers $= 2^{26}$

Value of sum of any subset $\leq 26 \cdot 10^6$

different subsets = 2^{26} = 67,108,864.

Values of sums: integers from 0 through 26×10^6

$f(\text{Subset of } S) \longrightarrow$ Sum of the elements in the subset.

Generalized Pigeonhole Principle:

$f: A \to B$ then there are at least

$\left\lceil \frac{|A|}{|B|} \right\rceil$ elements in A that map on to the same element in B

Class with 19 kids must have at least 10 boys or 10 girls. $\to \left\lceil \frac{19}{2} \right\rceil = 10.$

There are 677 classes offered by a college and 38 time periods.

How many rooms are required?

$$\left\lceil \frac{677}{38} \right\rceil$$

51 houses on a street with addresses from 1000 – 1099
$\Rightarrow$ 2 must have consecutive address numbers.

1000, 1001 | 1002, 1003 (x x) | – – | 1098, 1099.

50 buckets

smallest # n $\quad \left\lceil \frac{n}{m} \right\rceil = k.$

$\Rightarrow$ Placing n balls in m boxes.

In order to guarantee $\geq k$ in a box, need $n \geq \underbrace{1 + (k-1)m}$.

Why: if $n = (k-1)m$ could have $k-1$ in each box but then the next one would result in a box w/ $\geq k$ balls.

High school sign up for a committee.
Need ≥ 6 from one grade.

How many students need to sign up?

Boxes: grades : $m = 4$.
$k = 6$.

$$1 + (k-1)m = 1 + 5 \cdot 4 = 21.$$

Subsets with repetitions:

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Multi-set can have multiple copies of the same item:

$$\{1, 2, 2, 2, 3, 4, 4, 5\}.$$

(Subset w/ repetitions).

Selecting a dozen donuts.

Four varieties: chocolate, glazed, jelly, plain.
(order of varieties is fixed).

Donuts of the same variety are indistinguishable.
There is a large supply of each variety

How many ways to select a dozen donuts?
(order does not matter).

C C C G J J J J J J P P

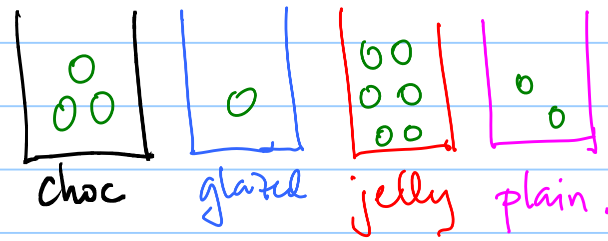
$$\left.\begin{array}{llll} 3 & \text{chocolate} & 6 & \text{jelly} \\ 1 & \text{glazed} & 2 & \text{plain} \end{array}\right\} \text{total of } 12.$$

\# ways to select a set of 12 items from 4 varieties = \# ways to put 12 identical balls into 4 distinguishable bins.
(no limit on \# per bin).

identical balls into distinguishable bins.

CCC G JJJJJJ PP

3 chocolate 6 jelly
1 glazed 2 plain $\Longleftrightarrow$



Bin for each variety.
balls in a bin = # donuts chosen for a particular variety.

Back to donut selection.

Binary encoding that uniquely specifies a particular donut selection.

Bijection:

order of varieties is fixed.

Donut Selection $\longleftrightarrow$ $\underbrace{\text{Binary code word}}$

will count the # of valid binary code words.

Binary code: #'s is (#varieties − 1)

1 — dividers

3 choc
1 glazed.
6 jelly.
2 plain.

000 1 0 1 000000 1 00

12 0's
3 1's.

$\left.\right\}\binom{15}{3}$ codewords

0000 1 1 0 1 0000000

4 choc
0 glazed.
1 jelly

7 plain.

10 identical prizes
to 100 different people. $\Longrightarrow$

10 identical balls.
100 Bins.

$\binom{109}{10}$

Codeword has 99 1's
10 0's.

Selecting 12 donuts. 4 varieties.

Constraint! at least 6 choc. donuts.

pick 6 choc
Now select 6 donuts from 4 varieties

$\binom{6+4-1}{4-1} = \binom{9}{3}$

Pick n items, m varieties $= \binom{n+m-1}{m-1} =$ # ways place n id balls into m distinct bins.

$n = 12$ identical choc bars
$m = 5$ kids

$\binom{12+5-1}{5-1} = \binom{16}{4}$

Each kid gets ≥ 1 choc bar.

~~Give~~ 1 to each kid.
7 left to distribute $\rightsquigarrow \binom{7+5-1}{5-1} = \binom{11}{4}$