

Recap: Selecting n items from a set of m varieties.

- Items of the same variety are indistinguishable.
- The order in which the items are selected does not matter.
- Unlimited quantity of each item to select from.

Picking a dozen donuts from 4 varieties.
(choc, glazed, jelly, plain).

Bijection between binary code words w/ $m-1$ 1's
and n 0's.

$$\hookrightarrow \binom{n+m-1}{m-1}$$

Added constraint: at least x of some variety.

(at least 4 chocolate).

$$\hookrightarrow \binom{8+4-1}{4-1} = \binom{11}{3}$$

Selecting 12 donuts of 4 varieties.

Bakery has short supply of chocolate (only 7 left).

By complement.

ways to select 12 donuts from 4 varieties (no restrictions)

$$\binom{12+4-1}{4-1}$$

ways to select 12 donuts from 4 varieties w/ 8 or more chocolates

$$\binom{4+4-1}{4-1}$$

varieties.

$12-8$

Number of solutions to the equation:

$$\Rightarrow X_1 + X_2 + X_3 + X_4 = 12$$

Each X_i must be a non-negative integer.

Bijection: Ways to select 12 donuts from 4 varieties



Solutions to the equation.

$X_i = \#$ selected from i^{th} variety.

$X_1 =$ $X_2 =$ $X_3 =$ $X_4 =$
 # choc # glazed # jelly # plain,

{
 4 choc
 0 glazed.
 3 jelly
 5 plain

 12

→

$X_1 = 4$
 $X_2 = 0$
 $X_3 = 3$
 $X_4 = 5$

 12.

$X_1 = 2$
 $X_2 = 5$
 $X_3 = 1$
 $X_4 = 4$

→

2 choc
 5 glazed
 1 jelly
 4 plain.

solutions to:

$$X_1 + X_2 + X_3 + \dots + X_m = n \quad \sim \text{# selected.}$$

variables

Each X_i is a non-negative integer.

↳
$$\binom{n+m-1}{m-1}$$

Permutations w/ Repetitions.

permutations of $S = \{H, E, L, P\}$.

HELP
HEPL
HLEP
HLPE
HPLE

HPEL
EHLP
:
etc.

permutations of $S = 4!$

What about permutations of $\{B, A, L, L\}$

↳ multi-set.
indistinguishable items.

permutations of B, A, L, \tilde{L} (how many?).

- BALL \tilde{L} >
- BA \tilde{L} L > BALL
BLA \tilde{L} >
B \tilde{L} AL > BLAL
BL \tilde{L} A > BLLA
B \tilde{L} LA >
:
:

Mapping from permutations of BALL \tilde{L}

to permutations of BALL

is ... (2-to-1)

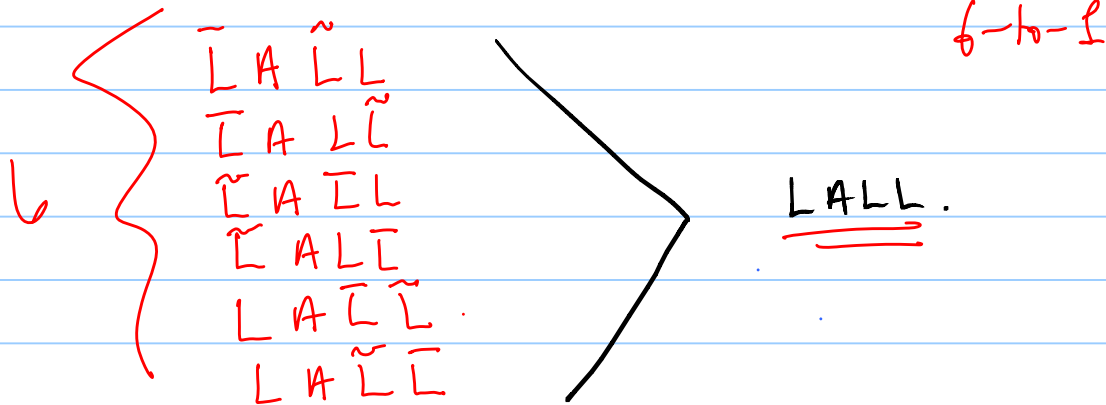
(# permutations of BALL)

=

(# permutations of BALL \tilde{L} / 2) = $\frac{4!}{2}$.

What about: $\{A, L, L, L\}$?

$A \bar{L} \bar{L} \tilde{L} \Rightarrow \# \text{ permutations.}$

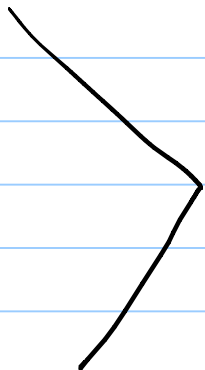


$$\frac{4!}{6} = \left(\frac{\# \text{ permutations of } \{A, \bar{L}, \bar{L}, \tilde{L}\}}{6} \right) = \# \text{ permutations of } \{A, L, L, L\}$$

What about: $\{A, L, L, L\}$?

$A \bar{L} \bar{L} \tilde{L} \Rightarrow \# \text{ permutations.}$

$\bar{L} A \tilde{L} L$
 $\bar{L} A L \tilde{L}$
 $\tilde{L} A \bar{L} L$
 $\tilde{L} A L \bar{L}$
 $L A \bar{L} \tilde{L}$
 $L A \tilde{L} \bar{L}$



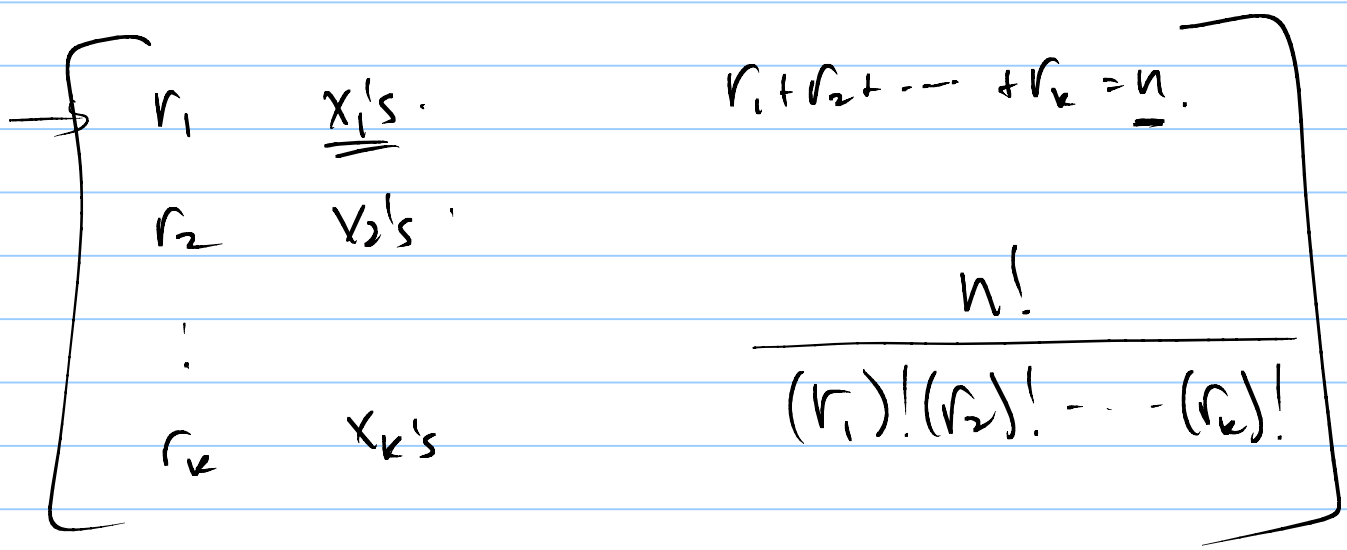
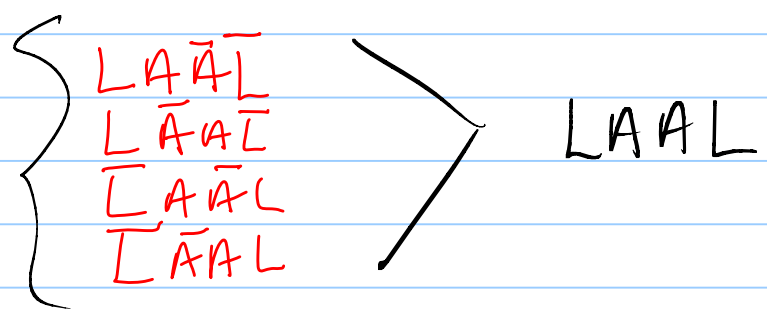
$LALL.$

permutations of $\{A, L, \bar{L}, \tilde{L}\}$

permutations
of $\{A, L, L, L\}$

A A L L

A \bar{A} L \bar{L}
4!



r_1 x_1 's = n
 r_2 x_2 's =
:
 r_k x_k 's =

places.

$$\frac{\binom{n}{r_1} \binom{n-r_1}{r_2} \binom{n-r_1-r_2}{r_3} \dots \binom{n}{r_k}}$$

M Y S S I S S I P P I
_{1 1 2 2 3 4 3 1 2 4}

$$\frac{11!}{4! 4! 2! 1!}$$

L's