Note Title 11/17/2014

Recap: Selecting n items from a set of m varieties.

- Items of the same variety are indistinguishable.
- The order in which the items are selected does not matter.
- Unlimited quantity of each item to select from.

Picking a dozen donuts from 4 varieties. (choc, glazed, jelly, plain).

Bijection between binary code words w/ $m-1$ 1's and n 0's.

$$\hookrightarrow \binom{n+m-1}{m-1}$$

Added constraint: at least x of some variety.

(at least 4 chocolate).

$$\hookrightarrow \binom{8+4-1}{4-1} = \binom{11}{3}$$

Selecting 12 donuts of 4 varieties.

Bakery has short supply of chocolate (only 7 left).

By complement.

\# ways to select 12 donuts from 4 varieties (no restrictions) − \# ways to select 12 donuts from 4 varieties w/ 8 or more chocolates

$$\left[\binom{12+4-1}{4-1} - \binom{4+4-1}{4-1}\right]$$

12−8

\# varieties.

Number of solutions to the equation:

$$\Rightarrow \quad X_1 + X_2 + X_3 + X_4 = 12$$

each X_i must be a non-negative integer.

Bijection: Ways to select 12 donuts from 4 varieties $\longleftrightarrow$ solutions to the equation.

X_i = \# selected from i^{th} variety.

$x_1 =$ # choc $\quad x_2 =$ # glazed $\quad x_3 =$ # jelly $\quad x_4 =$ # plain,

4 choc
0 glazed.
3 jelly
5 plain
12

$\rightarrow$

$x_1 = 4$
$x_2 = 0$
$x_3 = 3$
$x_4 = 5$
12.

$x_1 = 2$
$x_2 = 5$
$x_3 = 1$
$x_4 = 4$

$\rightarrow$

2 choc
5 glazed
1 jelly
4 plain.

solutions to:

$$x_1 + x_2 + x_3 + \cdots + x_m = n$$

varieties

selected.

each x_i is a non-negative integer.

$$\hookrightarrow \binom{n+m-1}{m-1}$$

Permutations w/ Repetitions.

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\# permutations of $S = \{H, E, L, P\}$.

HELP HPEL
HEPL EHLP
HLEP ⋮
HLPE etc.
HPLE

\# permutations of $S = 4!$

What about permutations of $\{B, A, L, L\}$ ↳ multi-set. indistinguishable items.

permutations of $B, A, L, \tilde{L}$ (how many?).

$$\left.\begin{matrix} -B\,A\,L\,\tilde{L} \\ -B\,A\,\tilde{L}\,L \end{matrix}\right\} \underline{BALL}$$

$$\left.\begin{matrix} B\,L\,A\,\tilde{L} \\ B\,\tilde{L}\,A\,L \end{matrix}\right\} BLAL$$

$$\left.\begin{matrix} B\,L\,\tilde{L}\,A \\ B\,\tilde{L}\,L\,A \end{matrix}\right\} BLLA$$

⋮ ⋮

Mapping from permutations of $BAL\tilde{L}$ to permutations of BALL is ... (2-to-1)

$$\left(\begin{matrix}\#\text{ permutations}\\ \text{of BALL}\end{matrix}\right) = \left(\frac{\#\text{ permutations of } BAL\tilde{L}}{2}\right) = \frac{4!}{2}.$$

What about: $\{A, L, L, L\}$?

$A\ L\ \bar{L}\ \tilde{L} \Rightarrow$ # permutations.

6-to-1

$$6\left\{\begin{array}{l} \bar{L}\ A\ \tilde{L}\ L \\ \bar{L}\ A\ L\ \tilde{L} \\ \tilde{L}\ A\ \bar{L}\ L \\ \tilde{L}\ A\ L\ \bar{L} \\ L\ A\ \bar{L}\ \tilde{L} \\ L\ A\ \tilde{L}\ \bar{L} \end{array}\right. \quad \Big\rangle \quad \underline{\underline{LALL}}.$$

$$\frac{4!}{6} = \left(\frac{\#\text{ permutations of } \{A, L, \bar{L}, \tilde{L}\}}{6}\right) = \#\text{ permutations of } \underline{\{A, L, L, L\}}$$

What about : $\{A, L, L, L\}$?

$A\ L\ \bar{L}\ \tilde{L} \Rightarrow$ # permutations.

$\bar{L}\ A\ \tilde{L}\ L$

$\bar{L}\ A\ L\ \tilde{L}$

$\tilde{L}\ A\ \bar{L}\ L$

$\tilde{L}\ A\ L\ \bar{L}$

$L\ A\ \bar{L}\ \tilde{L}$

$L\ A\ \tilde{L}\ \bar{L}$

LALL.

permutations of $\{A, L, \bar{L}, \tilde{L}\}$

permutations of $\{A, L, L, L\}$

A A L L

$A \bar{A} \quad L \bar{L}$

$\underbrace{\qquad\qquad}_{4!}$

$\left\{ \begin{array}{l} L A \bar{A} \bar{L} \\ L \bar{A} A \bar{L} \\ \bar{L} A \bar{A} L \\ \bar{L} \bar{A} A L \end{array} \right\} \to LAAL$

r_1 x_1's.

r_2 x_2's.

$\vdots$

r_k x_k's

$r_1 + r_2 + \cdots + r_k = n$.

$$\frac{n!}{(r_1)!(r_2)!\cdots(r_k)!}$$

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ n$

places.

r_1 x_1's $= n$

r_2 x_2's $=$

$\vdots$

r_k x_k's $=$

$$\binom{n}{r_1}\binom{n-r_1}{r_2}\binom{n-r_1-r_2}{r_3}\cdots\binom{r_k}{r_k}$$

$M I_1 S_1 S_2 I_2 S_3 S_4 I_3 P_1 P_2 I_4$

$$\frac{11!}{4!\,4!\,2!\,1!}$$

S's