Permutations w/ Repetitions.

Note Title 11/19/2014

MISSISSIPPI

I S I M S I I S S P P

Select places for S's. — $\binom{11}{4}$
Select places for I's. — $\binom{7}{4}$
Select places for P's. — $\binom{3}{2}$
Place final M.

perms:

$$\binom{11}{4}\binom{7}{4}\binom{3}{2}$$

$$\frac{11!}{\cancel{7!}\,4!}\cdot\frac{\cancel{7!}}{4!\,\cancel{3!}}\cdot\frac{\cancel{3!}}{2!\,1!}$$

$$\frac{11!}{4!\,4!\,2!\,1!}$$

MISSISSIPPI

perms = $11!/k$

$S_1 I_1 S_2 P_1 P_2 I_2 I_3 M S_3 I_4 S_4$ → SISPPIIMSIS

S.I.S.P.P.I.I. M.S.I.S →

$k = 4!\cdot 4!\cdot 2!\cdot 1!$ ← k-to-1 mapping

$$\frac{11!}{4!\,4!\,2!\,1!}$$

Note Title 11/18/2014

# of strings over $\{a, b, c, d\}$.

→ 6 a's
→ 5 b's
→ 2 c's
→ 9 d's.

$$\frac{22!}{6!\,5!\,2!\,9!}$$

Assignment of 20 different jobs to five processors. A, B, C, D, E

Each processor gets same number of jobs. $\frac{20!}{(4!)^5}$

4 of each letter A...E.

A	E					B
1	2	3	4	...		20

No restrictions.

$$\underbrace{5 \cdot 5 \cdot 5 \cdots \cdot 5}_{20} = 5^{20}.$$

Give cookies to 10 kids.

Each gets 1 cookie

3 choc chip
1 sugar
4 ginger
2 oatmeal raisin.

C	C	S	G	G	O	O	C	G	G
1	2								10

$$\frac{10!}{3!\,1!\,4!\,2!}$$

Plan schedule for work outs. 20 days.

10 run

5 bike   # schedules. $\frac{20!}{(10)!\,5!\,5!}$

5 swim

Balls into Bins. n balls
m distinct bins.

Balls are distinguishable.
No restrictions.

$$\underbrace{m \cdot m \cdot m \cdots \cdot m}_{n \text{ times}} = m^n$$

B3	B1	—	Bm	B2
1	2			n

r_1 in bin 1
r_2 in bin 2.
$\vdots$
r_m in bin m

$$r_1 + r_2 + \cdots + r_m = n.$$

$$\frac{n!}{(r_1)!\,(r_2)!\cdots(r_m)!}$$

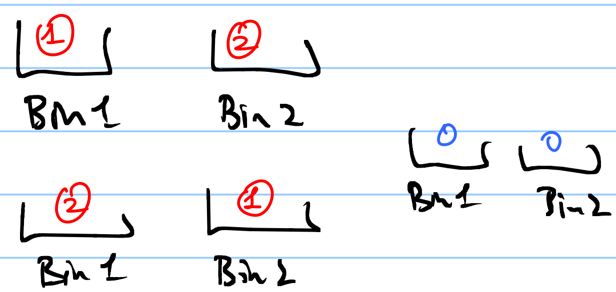
Same # in each bin n/m

$$r_1 = r_2 = \cdots = r_m = n/m$$

$$\frac{n!}{[\,(n/m)!\,]^m}$$

Indistinguishable balls:
No restrictions.

$$\binom{n+m-1}{m-1}$$



Same # in each bin: 1.

for $i = 1$ to n.
  for $j = i$ to n.
    for $k = j$ to n.
      [*
    end
  end
end

times * reached

$=$ # triplets.

$1 \le i \le j \le k \le n$.

(i, j, k)

$$X_1 = i - 1 \qquad X_1 \ge 0$$
$$X_2 = j - i \qquad X_2 \ge 0$$
$$X_3 = k - j \qquad X_3 \ge 0$$
$$X_4 = n - k. \qquad X_4 \ge 0$$

$$X_1 + X_2 + X_3 + X_4 = n - 1$$

solutions $\rightarrow \binom{n-1+4-1}{4-1}$

Yahtzee

Roll A Die 5 times: each roll is $\{1,2,3,4,5,6\}$

order matters:

1-2-3-4-5 $\neq$ 5-1-2-3-4

$\Rightarrow$ # rolls. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$

$\Rightarrow$ # rolls w/ a large straight (can be rearranged to be all consecutive).

2-1-5-4-3

6-4-3-2-5

1-2-6-4-5

1-5
2-6 $\Big\}$ $2 \cdot 5!$

$\Rightarrow$ at least two have same value.

$6^5 - [\underbrace{\text{\# rolls all distinct.}}_{}]$ P(6,5)

$\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$ $\underline{\ \ }$
6 5 4 3 2

⇒ # of ≥ 4 of a kind.

exactly 4 same

exactly 5 same → 6

$$\left(6 \cdot \binom{5}{4} \cdot 5\right)$$

2 2 $\cancel{*}$ 2 2

# of outcomes that happen four times.

$$\left[6 \cdot \binom{5}{4} \cdot 5 + 6\right]$$

⇒ Full House.

_ 3 _ _ 3

2 of a kind

$$\left[6 \cdot \binom{5}{2} \cdot 5\right]$$

⇒ From a set of 10 distinct 2-digit #'s. (natural #'s).
2 <u>non-empty</u> subsets will have the same sum.