Note Title 11/26/2014

Recap: Algebraic functions to represent sequences:

$$f_0, f_1, f_2, \ldots$$

$$F(x) = f_0 + f_1 x + f_2 x^2 + \ldots$$

Sequence: S_k = # ways to select a k-set of items from a particular set.

Infinite set of one variety: $\{S_n\}$ $1, 1, 1, \ldots$

$$F(x) = 1 + x + x^2 + \ldots = \frac{1}{1-x}$$

Finite # of one variety. (e.g. m apples) $1, 1, \ldots, 1$ $(m+1)$ times.

$$F(x) = 1 + x + \ldots + x^m = \frac{1-x^{m+1}}{1-x}.$$

Infinite supply of two varieties: $1, 2, 3, \ldots$

$$F(x) = 1 + 2x + 3x^2 + \ldots = \frac{1}{(1-x)^2}$$

Infinite supply of items, bundled into groups of k

$$1, \underbrace{0, 0, \ldots, 0}_{k-1 \text{ 0's}}, 1, 0, 0, \ldots, 0, 1, \ldots$$

$$F(x) = 1 + x^k + x^{2k} + \ldots = \frac{1}{1-x^k}.$$

From last time:

1 Banana

2 Apples

$$F(x) = 1 + 2x + 2x^2 + x^3$$

Two apples
Two bananas.

S_0	S_1	S_2	S_3	S_4
1	2	3	2	1
		AA	AAB	
		AB	ABB	
		BB		

$$1 + 2x + 3x^2 + 2x^3 + x^4$$

3 apples: $A(x) = 1 + x + x^2 + x^3$

(then add 1 3-pack of oranges).

2 Bananas: $B(x)$. $1 + x + x^2$.

Pool 3 apples + 2 Bananas into one set.

How many ways to select k items from the pooled set.

S_0 S_1 S_2 S_3 S_4 S_5

$$A(x)B(x) = (1 + x + x^2)(1 + x + x^2 + x^3).$$

$S_1(x)$: generating function for selecting from set S_1.

$S_2(x)$: generating function for selecting from set S_2

If S_1 and S_2 have no common varieties then:

$S_1(x) \cdot S_2(x)$ is the generating function for selecting from $S_1 \cup S_2$.

Select from:

3 apples. $\rightarrow 1+x+x^2+x^3 = \frac{1-x^4}{1-x}$

1 banana. $\rightarrow 1+x$

4 packs of oranges. $\rightarrow 1+x^4+x^8+\cdots = \frac{1}{1-x^4}$

2 packs of kiwi. $\rightarrow 1+x^2+x^4+\cdots = \frac{1}{1-x^2}$

$$\frac{1-\cancel{x^4}}{1-x} \cdot (1+x) \cdot \frac{1}{\cancel{1-x^4}} \cdot \frac{1}{1-x^2} = \frac{(1+x)}{(1-x)\underbrace{(1-x^2)}_{*}}$$

$$= \frac{\cancel{1+x}}{(1-x)\underbrace{\cancel{(1+x)}(1-x)}_{*}} = \frac{1}{(1-x)^2}$$

$$= 1 + 2x + 3x^2 + \cdots$$

$\#$ ways of putting to gether n cents worth of stamps from an inf collection of

3¢ stamps

11¢ stamps.

7¢ stamps.

3 cents: $1+x^3+x^6+\cdots = \dfrac{1}{1-x^3}$

7 cent: $\dfrac{1}{1-x^7}$ 11 cent: $\dfrac{1}{1-x^{11}}$

$$S(x) = \frac{1}{(1-x^3)(1-x^7)(1-x^{11})}$$

ways to select seven donuts from 3 varieties if

- No more than four of any variety is chosen?

$$(1+x+x^2+x^3+x^4)^3$$ ⤳ coefficient of x^7.

Binomial Theorem: (alternative view).

Pick subset of k from

$\{a_1, a_2, a_3, \dots, a_n\}$.

Generating function for ways to select subsets from $\{a_i\}$

$G_i(x) = 1 + x$

$\{a_1\}$ $\{a_2\}$ - - - - $\{a_n\}$.

$\{a_1, a_2, \dots, a_n\} = G_1(x)\,G_2(x) \cdots G_n(x)$

$\binom{n}{k} x^k + \quad = (1+x)^n$

$\binom{n}{0} 1 + \binom{n}{1} \cdot x + \binom{n}{2} x^2 + \cdots + \binom{n}{n} x^n = (1+x)^n$

$\binom{n+m-1}{m-1}$ $\qquad$ $\dfrac{n!}{(r_1)!\,(r_2)! \cdots (r_k)!}$