Discrete Probability.

Note Title 12/1/2014

- Algorithms make random choices. → running time, behavior/output.
- Testing computer systems
  generate input/demand at random.
- Model structures (internet, social networks)
  by random process.

Experiment that can result in one out of a set of possible outcomes.

Sample space = set of all possible outcomes

- Poker hand from a standard card deck. $\binom{52}{5}$
- Roll blue & red die. (5, 2)

$S = \{1,2,3,4,5,6\}^2$ $|S| = 36$.

- Flip a coin 3 times.

{HHH, HHT, ..., TTT} $|S| = 2^3$.

An Event E is a subset of S, the Sample space.

Dice: The two dice have the same number.

{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

Coin: There are more heads than tails.

{HHH, HHT, HTH, THH}

Probability over the outcomes of an experiment.

$p: S \to \mathbb{R}$. $0 \le p(x) \le 1$ $\forall x \in S$ $\sum_{x \in S} p(x) = 1$.

(S: sample space)

Roll of a single die: $S = \{1, 2, 3, 4, 5, 6\}$.

$$p(1) = p(2) = p(3) = p(4) = p(5) = \frac{1}{7}.$$

$$p(6) = \frac{2}{7}$$

$$\text{prob}(E) = \sum_{x \in E} p(x).$$

$S = \{1, 2, 3, 4, 5, 6\}$.

E = event that outcome is even.

What is E? $\{2, 4, 6\}$

What is $p(E)$? $p(E) = p(2) + p(4) + p(6)$

$= \frac{1}{7} + \frac{1}{7} + \frac{2}{7} = \frac{4}{7}$.

Most common distribution is the uniform distribution:

$\forall x \in S$ $p(x) = \frac{1}{|S|}$.

If the distribution over S is uniform then.

$$p(E) = \sum_{x \in E} \frac{1}{|S|} = \frac{|E|}{|S|}$$

=

Roll blue + red die: uniform distribution.

What is prob of each event? B: Sum ≤ 3. $B = \{(1,1), (1,2), (2,1)\}$

$$P(B) = \frac{3}{36}$$

C: Red = Blue + 1

$$C = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$$

$$p(C) = 5/36.$$

D: Red < Blue. $\frac{15}{36}$.

(x, y) $y > x$.

2 - 6

$(\cdot, 1) \rightsquigarrow 5$
$(\cdot, 2)$ +4
$(\cdot, 3)$ +3
$(\cdot, 4)$ +2
$(\cdot, 5)$ +1

Flip coin 10 times: uniform distribution.

What is $|S|$? 2^{10}

Prob first + last flips are heads? $\frac{2^8}{2^{10}} = \frac{1}{2^2} = \frac{1}{4}$.

H ——— H
8

H = Prob ≥ 1 Heads: $\quad |H| = 2^{10} - 1$

$$P(H) = \frac{2^{10} - 1}{2^{10}} = 1 - \frac{1}{2^{10}}$$

$$P(H) = 1 - P(\overline{H}) = 1 - \frac{1}{2^{10}}$$

$\overline{H}$: #heads is 0.

Poker Hand: uniform distribution.

Prob full house: $\dfrac{13 \cdot \binom{4}{2} \cdot 12 \binom{4}{3}}{\binom{52}{5}}$

Prob 2 of a kind. $\dfrac{13 \cdot \binom{4}{2} \cdot \binom{12}{3} 4^3}{\binom{52}{5}}$

Prob ≥ 1 club.

$$1 - \text{Prob}(\text{no clubs}) = 1 - \frac{\binom{39}{5}}{\binom{52}{5}}$$