Note Title 12/1/2014

Complement of Events:

Experiment w/ Sample space S.

5-card hand from a standard playing deck.

$C \subseteq S$ an event

$\overline{C} \subseteq S$ is the complement of event C.

C: Hand has ≥ 1 club.

$\overline{C}$: Hand has no clubs.

$$\text{Prob}[C] = 1 - \text{Prob}[\overline{C}].$$

(This is true for non-uniform distributions, but we will show it for uniform distributions)

Perfectly shuffled deck = uniform distribution.

$$\text{Prob}[C] = \frac{|C|}{|S|} = \frac{|S| - |\overline{C}|}{|S|} = \frac{|S|}{|S|} - \frac{|\overline{C}|}{|S|}$$

$$= 1 - \text{prob}(\overline{C}).$$

$$\text{Prob}(\overline{C}) = \frac{\binom{39}{5}}{\binom{52}{5}} \qquad \text{Prob}(C) = 1 - \frac{\binom{39}{5}}{\binom{52}{5}}$$

Same idea for inclusion - exclusion principle:

"Probability of condition 1 or condition 2"

$\hookrightarrow$ Inclusive or: "or both"

Example: Hand has a pair of 8's or a pair of queens

A_1: Set of outcomes for which condition 1 holds. (pair of 8's)

A_2: Set of outcomes for which condition 2 holds. (pair of queens)

$$P[A_1 \cup A_2] = \frac{|A_1 \cup A_2|}{|S|} = \frac{|A_1| + |A_2| - |A_1 \cap A_2|}{|S|} =$$

$$= \frac{|A_1|}{|S|} + \frac{|A_2|}{|S|} - \frac{|A_1 \cap A_2|}{|S|} = p(A_1) + p(A_2) - p(A_1 \cap A_2)$$

Prob[Pair of 8's or a Pair of Q's]

P(Pair of 8's) + P(Pair of Q') − P(pair of 8's & pair of queens)

$$2\frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} - \frac{\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}}$$

Conditional Probability & Independence.

Note Title 12/3/2014

Experiment: Flip a coin 3 times. $P(B|A)$

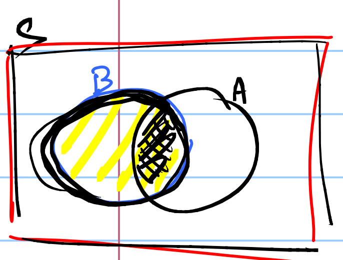
S:
HHH
HHT
HTH
HTT
~~THH~~
~~THT~~
~~TTH~~
~~TTT~~

A: First flip comes up heads.

B: There are at least 2 Heads.

C: # heads is odd.

Conditional Probability: $P(A|B)$ "Probability of A given B"



$$= \frac{P(A\cap B)}{P(B)}$$

↳ for uniform distributions: $= \dfrac{|A\cap B|/|S|}{|B|/|S|} = \dfrac{|A\cap B|}{|B|}$

Events A & B are independent if $\boxed{P(A|B) = p(A).}$ ①

if: $\boxed{\dfrac{p(A\cap B)}{p(B)} = p(A)}$ $\Longleftrightarrow$ $p(A\cap B) = p(A)p(B)$ ②

$p(A|B)$

$\Longleftrightarrow$ $p(B) = \dfrac{p(A\cap B)}{p(A)} = p(B|A)$ ③

A : First flip comes up heads. $\longrightarrow$ $P(A) = 1/2$

B : There are at least 2 Heads. ~ $P(B) = 4/8 = 1/2$.

HHH HHT HTH THH

C : # heads is odd. $P(C) = \frac{4}{8} = 1/2$.

{ HTT, THT, TTH, HHH }

$$P(B|A) = \frac{|B \cap A|}{|A|} = \frac{3}{4} \neq P(B)$$

A & B are not indep.

$$P(B|C) = \frac{1}{4} = \frac{1}{4} \neq P(B)$$

$$P(A|C) = \frac{2}{4} = \frac{1}{2} = P(A)$$

$P(A \cap C) = \frac{2}{8}$ $\qquad$ $P(A) \cdot P(C) = \frac{1}{2}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(C|A) \cdot$

$P(C) \cdot$

Select a group of four kids at random from a class of 20 kids (10 boys + 10 girls).

C = Charlie picked. $= \dfrac{\binom{19}{3}}{\binom{20}{4}} = P(C) = P(A).$

A = Angela picked

Charlie + Angela picked: $P(C \cap A) = \dfrac{\binom{18}{2}}{\binom{20}{4}}$

$P(C) \cdot P(A) = P(C \cap A).$

$$\frac{\binom{19}{3}}{\cancel{\binom{20}{4}}} \left[\frac{\binom{19}{3}}{\cancel{\binom{20}{4}}} \right] = \frac{\binom{18}{2}}{\cancel{\binom{20}{4}}} \quad \binom{20}{4}$$

$$\frac{\cancel{19} \cdot \cancel{18} \cdot \cancel{17} \cdot 19 \cdot \cancel{18} \cdot \cancel{17}}{3 \cdot \cancel{2} \quad \cancel{3} \cdot \cancel{2}} = \frac{\cancel{18} \cdot \cancel{17} \cdot \quad 20 \cdot \cancel{19} \cdot \cancel{18} \cdot \cancel{17}}{\cancel{2} \quad 4 \cancel{3} \cdot 2 \cdot 1}$$

$$\frac{19}{3} \overset{?}{=} \frac{20}{4}.$$

Charlie picked. $\binom{19}{3} / \binom{20}{4}$

Same # girls + boys. $\binom{10}{2}\binom{10}{2} / \binom{20}{4}$

Pr (Charlie picked | Same # girls + boys) $= \dfrac{|\text{Charlie + Same \#}|}{|\text{Same \#}|}$ $\quad \dfrac{9 \binom{10}{2}}{\binom{10}{2}\binom{10}{2}}$ ✓

$$\frac{9 \cancel{\binom{10}{2}} \cdot 1}{\binom{10}{2} \cancel{\binom{10}{2}}} = \frac{\binom{19}{3}}{\binom{20}{4}}$$

$\binom{10}{2} = \dfrac{10 \cdot 9}{2 \cdot 1}$

$$\frac{\cancel{9} \cdot 2 \cdot 1}{10 \cdot \cancel{9}} = \frac{\cancel{19} \cdot \cancel{18} \cdot \cancel{17}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \quad \frac{4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{20 \cdot \cancel{19} \cdot \cancel{18} \cdot \cancel{17}}$$

$$\| \quad 1/5 = \frac{4}{20} = \frac{1}{5}.$$

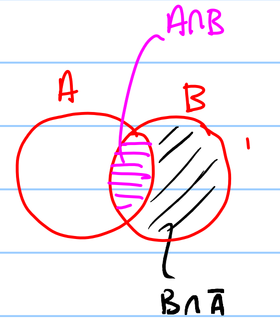
10 identical prizes to 20 kids.
random distribution.

$n = 10$
$m = 20.$

$$\binom{20+10-1}{20-1} = \binom{29}{19} = \binom{29}{10}$$

A: Someone gets more than one.
B: $\geq$ One goes to Charlie & $\geq$ one goes to Ann.

$$P(A) = 1 - \frac{\binom{20}{10}}{\binom{29}{10}}$$



$A \cap B = B - (B \cap \bar{A}).$

$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{|B| - |B \cap \bar{A}|}{|B|}$$

$$= \frac{\binom{8+20-1}{20-1} - \binom{18}{8}}{\binom{8+20-1}{20-1}}$$

Give one to Charlie & Angela then distribute remaining 8 to the 20 kids w/ no restrictions.

$B \cap \bar{A}$: Charlie & Angela get ≥ 1 and no kid gets more than one.
Give one to Charlie & Angela.
Now select which 8 kids from the remaining 18 get a prize.

$$\binom{18}{8}$$