n balls into m distinguishable bins:

$00|0|000$

$\Downarrow$

	Distinguishable balls	Indistinguishable (= identical) Balls.
No restrictions	m^n	$\binom{n+m-1}{m-1}$
At most one ball per bin. $(n \le m)$	$m \cdot (m-1) \cdots (m-n+1)$ $P(m,n)$.	$\binom{m}{n}$
Specific # in each bin. r_i in bin i. $r_1+r_2+\cdots+r_m=n$.	$\frac{n!}{(r_1)!(r_2)!\cdots(r_m)!} = \binom{n}{r_1}\binom{n-r_1}{r_2}\cdots\binom{r_m}{r_m}$	1.

If same # balls in each bin then it must be that m evenly divides n. Then $\frac{n}{m}$ in each bin $(r_1 = r_2 = \cdots = r_m = \frac{n}{m})$.

Distinguishable balls into distinguishable bins (no restrictions).

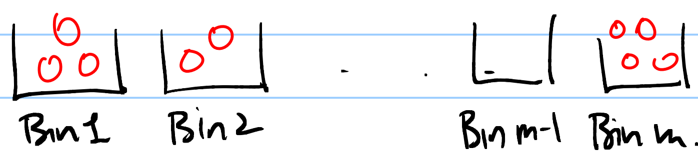
m choices for each position. j^{th} position indicates which bin for ball j

m^n

$\underline{\quad}_1 \quad \underline{\quad}_2 \quad \underline{\quad}_3 \quad \cdots \quad \underline{\quad}_n$.

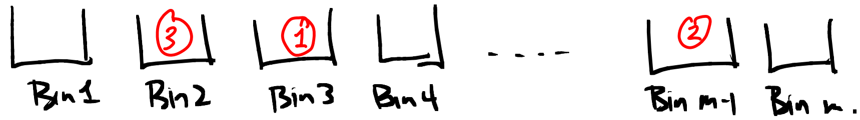
- # strings of length n over an alphabet with m characters.
- Schedule planning: n days, choose one of m activities per day.
- Assignment of n different people to m different projects (no restrictions on # people in each project).

Indistinguishable Balls, Distinguishable Bins (no restrictions).



$n \le m$ or $n \ge m$

- Selecting n items from m varieties (large supply of each variety).
  # balls in bin j = # chosen from variety j.
- Solutions to: $X_1 + X_2 + \cdots + X_m = n$
  X_i's are non-negative integers.
  X_j = # balls in bin j.
- n identical jobs assigned to m different processors.



n distinguishable balls into m distinguishable bins.
At most one per bin. Need $m \geq n$.

choice of bin for each ball

___ ___ ___ . . . . ___

1 2 3 n.

an n-permutation from a set of size m.

No repetitions: $m(m-1) \cdots (m-n+1) = P(m,n)$.

- Distribute n different prizes to m different people at most one per person.
- Select class officers from an 8^{th} grade class. Order of selection matters:

___ ___ ___ ___

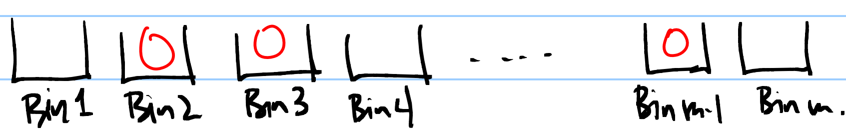
Pres VP Treas Secretary.

- Schedule of n days m choices for each day — no repetitions.

n indistinguishable balls into m distinguishable bins.
At most one per bin. Need $m \geq n$.

- Subset of size n from a set of size m.

$\binom{m}{n}$



Which of the n bins get balls?

- Select a committee of size n from a club w/ m members. (No special roles)
- # binary string with m bits and n 1's

n distinguishable balls into m distinguishable bins. r_j: balls in bin j.

Must have $r_1 + r_2 + r_3 + \cdots + r_m = n$.

↳ total # balls.

Bin selection for each ball.

$$\underset{1}{\underline{\ 1\ }}\quad \underset{2}{\underline{\quad}}\quad \underset{3}{\underline{\ 2\ }}\quad \underset{4}{\underline{\ 1\ }}\quad \cdots \quad \underset{n.}{\underline{\ 2\ }}$$

r_1 1's
r_2 2's
⋮
r_m m's.

$$\frac{n!}{(r_1)!\,(r_2)!\cdots(r_m)!}$$

- Placement of people in offices. Each office has a capacity. Total capacity = total # of people.
- Schedule choice of m activities per day. n days total. r_j = # times activity j occurs in the schedule.

n indistinguishable balls into m distinguishable bins. r_j: balls in bin j.

Must have $r_1 + r_2 + r_3 + \cdots + r_m = n$.

↳ total # balls.

- 10 candy bars to 5 kids. (candy bars all same) Same # to each kid.

$\Rightarrow$ 2 to each kid.