**n balls into m distinguishable bins:**

<table>
<thead>
<tr>
<th>Distinguishable balls</th>
<th>Indistinguishable (≡ identical) Balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reasonable</td>
<td>$m^n$</td>
</tr>
<tr>
<td>At least one ball per</td>
<td>$m \cdot (m-1) \cdots (m-n+1)$</td>
</tr>
<tr>
<td>bin. $(n \leq m)$</td>
<td>$P(m,n)$</td>
</tr>
<tr>
<td>Specific # in each bin, $r_i$ in bin $i$, $r_1 + r_2 + \cdots + r_m = n$</td>
<td>$\frac{n!}{(r_1)! (r_2)! \cdots (r_m)!} = \frac{(n)}{(r_1) (r_2) \cdots (r_m)} \cdot \frac{(r_1) (r_2) \cdots (r_m)}{n!}$</td>
</tr>
</tbody>
</table>

If same # balls in each bin then it must be that m evenly divides n. Then $\frac{n}{m}$ in each bin ($r_1 = r_2 = \cdots = r_m = \frac{n}{m}$).
Distinguishable balls into distinguishable bins (no restrictions).

\[ m \text{ choices for each position. } i^{th} \text{ position indicates which bin for ball } j. \]

\[ 1 \ 2 \ 3 \ \ldots \ n. \]

- \# strings of length \( n \) over an alphabet with \( m \) characters.
- Schedule planning: \( n \) days, choose one of \( m \) activities per day.
- Assignment of \( n \) different people to \( m \) different projects (no restrictions on \# people in each project).

Indistinguishable Balls, Distinguishable Bins (no restrictions).

\[ \begin{array}{cc}
\hline
0 & 0 \\
- & - \\
\hline
\text{Bin 1} & \text{Bin 2} \\
\text{Bin m-1} & \text{Bin m} \\
\end{array} \]

\[ n \leq m \]

or \[ n \geq m \]

- Selecting \( n \) items from \( m \) varieties (large supply of each variety).

\[ \text{# balls in bin } j = \text{# chosen from variety } j. \]

- Solutions to:

\[ x_1 + x_2 + \ldots + x_m = n \]

\( x_i \)'s are non-negative integers.

\[ x_j = \text{# balls in bin } j. \]

- \( n \) identical jobs assigned to \( m \) different processors.
N distinguishable balls into m distinguishable bins.
At most one per bin. Need \( m \geq n \).

- Choose 1 bin for each ball (1 2 3 \ldots N).
- No repetitions: \( m(m-1) \ldots (m-n+1) = P(m,n) \).

- Distribute different prizes to \( m \) different people at most one per person.
- Select class officers from an 8th grade class.
  - Order of selection matters:
    - President, Vice President, Treasurer, Secretary.
- Schedule of \( n \) days in chores for each day — no repetitions.

N indistinguishable balls into m distinguishable bins.
At most one per bin. Need \( m \geq n \).

- Subset of size \( n \) from a set of size \( m \).

\[ \binom{m}{n} \]

Which of the \( n \) bins get balls?

- Select a committee of size \( n \) from a club of \( m \) members. (No special roles)
- # binary strings with \( m \) bits and \( n \) 1's
N distinguishable balls into m distinguishable bins. \( r_j \) balls in bin \( j \).

Must have \( r_1 + r_2 + r_3 + \cdots + r_m = N \).

Corresponding bin selection:

\[
\begin{array}{cccccc}
1 & 2 & 1 & & & 2 \\
1 & 2 & 3 & 4 & & \cdots & n.
\end{array}
\]

\[
\begin{align*}
r_1 & \text{: 1's} \\
r_2 & \text{: 2's} \\
r_m & \text{: m's.}
\end{align*}
\]

\[
\frac{n!}{(r_1)! (r_2)! \cdots (r_m)!}
\]

- Placement of people in offices. Each office has a capacity. Total capacity = total # of people.
- Schedule choice of m activities per day. \( n \) days total. \( r_j \) times activity \( j \) occurs in the schedule.

N indistinguishable balls into m distinguishable bins. \( r_j \) balls in bin \( j \).

Must have \( r_1 + r_2 + r_3 + \cdots + r_m = N \).

\[
\begin{align*}
\text{10 candy bars to 5 kids.} & \quad \text{(Candy bars all same)} \\
\text{Same # to each kid.} & \quad \Rightarrow 2 \text{ to each kid.}
\end{align*}
\]