**ICS 6D** **Due: Wednesday, October 15, 2014, 3:00PM**

# Homework 1

*Instructor: Sandy Irani*

1. Give a truth table for the following logical expession: $(p \vee \neg r) \rightarrow \neg q$.
2. The propositional variables $p$, $q$, and $r$ have the following truth values:

   - $p = T$
   - $q = F$
   - $r = F$

   What is the truth value of the following compound propositions:

   (a) $(p \wedge q) \rightarrow \neg r$
   (b) $\neg(r \vee \neg p \vee \neg q)$
   (c) $\neg(q \wedge r) \rightarrow \neg p$
   (d) $\neg(q \wedge r) \vee (p \wedge \neg r)$
3. Which statements below evaluate to true?

   (a) If 3 is a prime number then 4 is also a prime number.
   (b) If January has 28 days, then 4 is a prime number.
4. Are the expressions $p \rightarrow q$ and $q \rightarrow p$ logically equivalent? Use a truth table to prove your answer.
5. Define the following propositions:

   - $s$: a person is a senior.
   - $v$: a person is at least seventeen years of age.
   - $p$: a person is allowed to park in the parking lot.

   Express each of the following English sentences with a logical expression:

   (a) A person is allowed to park in the parking lot only if they are a senior and at least seventeen years of age.
   (b) A person can park in the parking lot if they are a senior or at least seventeen years of age.
6. In this question the domain of discourse is a set of employees in a company. Define the following predicates:

   - $T(x)$: person $x$ is a member of the executive team.
   - $S(x)$: person $x$ received a large bonus.

   Express the following English sentences with quantified logical expressions:

   (a) Someone did not get a large bonus.
   (b) Everyone got a large bonus.
   (c) Sam did not receive a large bonus even though he is a member of the executive team.

(d) Someone who is not an executive received a large bonus.

(e) Every executive team member got a large bonus.

7. In this problem, the domain of discourse is the set of all integers. Which statements are true? If an existential statement is true, give an example. If a universal statement is false, give a counter-example.

   (a) $\exists x(x + x = 1)$

   (b) $\forall x(x^2 - x \neq 1)$

   (c) $\forall x(x^2 - x \neq 0)$

   (d) $\exists x(x + 2 = 1)$

8. Define the following sets:

   - $A = \{x \in \mathbb{Z} : x \text{ is even}\}$
   - $B = \{x \in R : x \geq 1\}$
   - $C = \{-3, 1, 2, 6, 7, 9\}$
   - $D = \{2, 3, 5, 9, 10, 17\}$

   Indicate whether the following statements are true or false:

   (a) $\pi \in B$

   (b) $A \subseteq B$

   (c) $C \subseteq B$

   (d) $8 \in A \cap B$

   (e) $A \cap C \subseteq B$

   (f) $C \subseteq A \cup B$

   (g) $A \cap C \cap D = \emptyset$

   (h) $|C| = |D|$

   (i) $|C \cap D| = 3$

9. Venn diagram question.

$$A \cap (B \cup C)$$

$$\overline{B} \cap (A \cup C).$$

10. Define the sets $X = \{a, b, c\}$ and $Y = \{1, 2\}$. Show the set $X \times Y$ by listing the elements with set notation.

11. Show the set $\{0, 1\}^3$ by listing the elements with set notation. You can specify each element as a string instead of including the parentheses and commas.

12. What is $|\{0, 1\}^2|$? What is $|\{0, 1\}^3|$?

13. For each of the functions below, indicate whether the function is onto, one-to-one or both. If the function is not one-to-one, give an example showing why.

   (a) $f : R \to R$. $f(x) = x^2$.

   (b) $g : R \to R$. $g(x) = x^3$.

   (c) $h : Z \to Z$. $g(x) = x^3$.

14. For each of the functions below, indicate whether the function is onto, one-to-one or both. If the function is not onto, specify its range by listing the elements using set notation. If the function is not one-to-one, give an example showing why.

    (a) $f : \{0,1\}^4 \to \{0,1\}^3$. The output of $f$ is obtained by taking the input string and dropping the first bit. For example, $f(1011) = 011$.

    (b) $g : \{0,1\}^3 \to \{0,1\}^3$. The output of $g$ is obtained by replacing the first bit with 1 regardless of whether the first bit is a 0 or a 1. For example, $g(001) = 101$ and $g(110) = 110$.

    (c) $h : \{0,1\}^3 \to \{0,1\}^3$. The output of $h$ is obtained by reversing the bits. For example $h(001) = 100$ and $h(110) = 011$.

15. Let $f$ be a function $f : \{0,1\}^2 \to \{0,1\}^3$. Is it possible that $f$ is a bijection? Why or why not?

16. Express the following sums using summation notation:

    (a) $(-2) + (-1) + 0 + 1 + 2 + 3 + 4 + 5$

    (b) $2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$

    (c) $0^3 + 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \cdots + (17)^3$.

    (d) The sum of the squares of the odd integers between 0 and 100.

17. Indicte whether the following equality is true and justify your answer:

$$\sum_{j=0}^{100} j^3 = \sum_{j=1}^{100} j^3$$

18. Give the first six terms of the following sequences. You can assume that the sequences start with an index of 1.

    (a) The $n^{th}$ term is $n^2$.

    (b) The first term is 1 and the second term is 2. The rest of the terms are the product of the two preceding terms.

    (c) $g_1 = 2$ and $g_2 = 1$. $g_n = n \cdot g_{n-2} + g_{n-1}$.

    (d) A geometric sequence in which the first value is 3 and the common ratio is 2.

    (e) An arithmetic sequence in which the first value is 3 and the common difference is 5.

19. Evaluate the following summations:

    (a) $\sum_{j=-2}^{2} j^3$

    (b) $\sum_{k=0}^{3} 2^k$