

A proposition is a statement that is either true or false.

Examples : $2 + 3 = 7$

$$5 - 2 = 3$$

7 is a prime number

Today is Friday

It will rain tomorrow

Vanilla is the best flavor of ice cream.

Not propositions :

How are you?

Eat your Vegetables.

Logical Variables : (P, q, r, ...)

denote an arbitrary proposition
truth value can be true or false

true : $P = T$

false : $P = F$.

like variables from algebra
except value is T or F,
instead of a number.

Logical operations can be used to combine propositions to get compound propositions.

Conjunction "AND" Symbol \wedge

If p & q are propositional variables,

$p \wedge q$ is a proposition.

Truth value of $p \wedge q$ depends on truth values of p and q .

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table shows
truth value of
 $p \wedge q$ for every
possible truth value
for p and q .

p : Sam is poor
 q : Sam is happy

Different ways to express $p \wedge q$ in English:

Sam is poor, but he is happy.

Sam is poor and happy

Although Sam is poor, he is happy.

} $P \wedge q$

Disjunction: $p \vee t$ "p or t"

p	t	$p \vee t$
T	T	T
T	F	T
F	T	T
F	F	F

(Inclusive OR)

Ambiguity in English:

Tonight I will go to the party or I will go to a movie.

Exclusive OR

The patient has high blood pressure or has a history of migraines.

Inclusive OR

Negation: $\neg p$.

p : it is raining today

$\neg p$: it is not raining today

it is not true that it is raining today

p	$\neg p$
T	F
F	T

Review:

$$p \wedge q \quad F$$

$$r \wedge p \quad T$$

$$q \vee r \quad T$$

$$p \vee r \quad T$$

$$\neg p \quad F$$

$$\neg q \quad T$$

Compound propositions can be built using one or more logical operations:

$$\begin{array}{c} p \vee \neg r \\ F \vee F = F. \end{array} \quad \begin{array}{l} p = F \\ r = T \end{array}$$

Need to specify the order in which operations are performed!

$$\begin{array}{c} (p \wedge q) \vee r \\ \text{underlined} \quad F \vee T = T. \end{array} \quad \begin{array}{l} p = F \\ q = T \\ r = T \end{array}$$

Order in which logical operations are applied:

1. \neg
2. \wedge
3. \vee

$$(\neg q) \vee r$$

$$(p \wedge q) \vee (\neg t)$$

Can override the default order with paren's:

$$\neg(\underline{q \vee r})$$

$r = T$
 $q = F.$

$$\neg T = F.$$

Good to include paren's as a reminder:

$$(p \wedge q) \vee r$$

=====

$$\neg p \vee (\underbrace{t \wedge r}_{\text{F} \vee F})$$

$p = T$
 $t = F$
 $r = T.$

$$\neg F$$

$$\neg(p \wedge t \wedge r)$$

$p = T$
 $t = T$
 $r = F$

$$\neg F$$

A truth table for a compound proposition shows the truth value for every possible combination of truth values for the propositional variables:

P	q	$\neg P$	$\neg P \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T
.	.	.	.

P	q	r	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \wedge r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

If a compound proposition has n variables the truth table has 2^n rows.

$$\begin{aligned} 2 \text{ variables} &\Rightarrow 4 \\ 3 \text{ variables} &\Rightarrow 8. \end{aligned}$$

• T p: $\pi > 3$

F q: 3 is a root of the equation $x^2 - 2 = 0$.

F r: The integer 5 is even.

$\neg r$ true.

T p v q

F F
F

q \wedge r

F

T (q \wedge r)

T

Conditional operation

 p, q propositions $p \rightarrow q$ false only when p is true
and q is false.

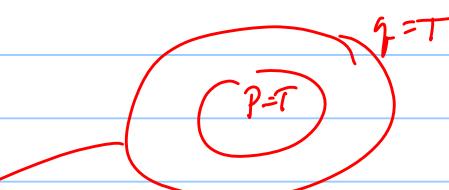
p	q	$p \rightarrow q$	$\neg p \vee q$	p is the hypothesis q is the conclusion
T	T	T	T	
F	F	F	F	
F	T	T	T	
F	F	T	T	

 $p \rightarrow q$ p : you study hard q : you will get an A $p \rightarrow q$ If you study hard, then
you will get an A.

Ways to express in English:

 $p \rightarrow q$.

- if p then q .
- if p, q
- P implies q .
- q , if p .
- p only if q
- p is sufficient for q .
- q is necessary for p .

 p : You have a driver's license. q : You are at least 16 years old.

From English to logic:

a : Joe's alarm went off.

s : Joe was on time for school.

Joe's alarm going off is a necessary condition for Joe to be on time for school.

$$S \rightarrow a$$

$$a \rightarrow S.$$

Joe's alarm going off is a ~~necessary~~^{sufficient} condition for Joe to be on time for school.

$$a \rightarrow S$$

If Joe was late for school then his alarm did not go off.

$$\neg S \rightarrow \neg a$$

Joe's alarm did not go off but he was still on time for school.

$$\neg a \wedge s$$

Joe was on time for school if his alarm went off.

$$a \rightarrow s$$

Joe was on time for school only if his alarm went off.

$$s \rightarrow a$$

Statement: $P \rightarrow q$.

Converse: $q \rightarrow P$

Contrapositive: $\neg q \rightarrow \neg P$

Inverse: $\neg P \rightarrow \neg q$.

P	q	$P \rightarrow q$	$\neg q \rightarrow \neg P$	$q \rightarrow P$	$\neg P \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	T	T	T

$\neg F \quad P \rightarrow T$

$P=F \quad q=T$

Statement:

If Sally joins the club, then Mildred will also join.

$$S \rightarrow M$$

Identify the converse, contrapositive inverse.

$$M \rightarrow S$$

$$\neg M \rightarrow \neg S$$

$$\neg S \rightarrow \neg M$$

If Sally does not join the club then Mildred won't either.

Inverse

If Mildred joins the club, then so will Sally.

Converse

If Mildred doesn't join the club then Sally won't join the club.

Contrapositive.

$p \leftrightarrow q$ means $(p \rightarrow q) \wedge (q \rightarrow p)$

p and q have the same truth value

P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Values for:

If 3 is prime then 4 is even. True.

If 3 is prime then 5 is even False.

T F

If 4 is prime then 5 is even. The

If 4 is prime then 4 is even. True.

$\pi > 3$ if and only if $\sqrt{5} > 2$. True