Note Title 1/16/2015

Predicates w/ more than one variable:

$$S(x,y)$$
$$Q(x,y,z)$$

To make into propositions, need to have a quantifier for each variable.

Nested Quantifiers of the Same type

Domain: set of people at a meeting.

$H(x,y)$: x shook y's hand.

$\forall x \forall y H(x,y)$ "Everyone shook everyone's hand".

Includes the case $x=y$!

"Everyone shook everyone else's hand."

$H(x,y)$

x \ y	Joe	Abe	Raj
Joe	T	T	T
Abe	T	F	T.
Raj	T	T	F

$\forall x \forall y \cancel{H}(x,y)$?

$P(x,y)$: x knows y's phone number.

$\exists x \, \exists y \, P(x,y)$

"Someone knows Someone's phone number."

could be satisfied by the case $x=y$.

$P(x,y)$

x \ y	Joe	Abe	Raj
Joe	F	F	F
Abe	F	T	F
Raj	F	F	T

"Someone knows Someone else's phone number."

$\exists x \, \exists y \, P(x,y)$?

Example: $x = y =$ Abe

Note Title 1/14/2015

Nested Quantifiers

$\exists x \, \forall y \; P(x,y)$ — Someone knows everyone's phone number.

$\forall x \, \exists y \; P(x,y)$ — Everyone knows someone's phone number.

Game between two players:

Existential Player	Universal Player
trying to make the predicate true	trying to make the predicate false.
Selects existentially quantified variables	Selects universally quantified variables.
Predicate true?	Predicate false?
Quant Statement is true	Quant statement false.

Domain = $\{1,2,3,4,5\}$

$\exists x \, \forall y \; P(x,y)$

Someone knows everyone's phone number.

$x=3$.

True for Group 1.

False for Group 2.

$x \backslash y$	1	2	3	4	5
1	F	F	F	F	F
2	F	T	F	F	T
3	T	T	T	T	T
4	F	T	T	T	T
5	T	T	T	T	F

Group 1.

$x \backslash y$	1	2	3	4	5
1	T	F	T	T	T
2	F	T	F	F	T
3	T	F	T	F	F
4	F	T	T	T	T
5	T	T	T	T	F

Group 2.

True for Gp 2.

$\forall x \, \exists y \; P(x,y)$

Everyone knows someone's phone number.

$x=1$, False for Gp 1

Domain: all real numbers.

$\forall x \, \exists y \quad x+y=0.$ T

$\exists x \, \forall y. \quad x+y=0.$ F

$\forall x \, \exists y \quad x \cdot y = 1.$ $x=0.$ False.

$\exists x \, \forall y \quad x \cdot y = 0.$ $x=0$ True.

$\forall x \, \exists y \, ((x=0) \vee x \cdot y = 1)$

True.

Domain = real #'s integers.

Domain = $\{1,2,3,4,5\}$

$\exists x \forall y \; P(x,y)$

Someone knows everyone else's phone number.

x \ y	1	2	3	4	5
1	F	F	F	F	F
2	F	T	F	F	T
3	T	T	T	F	T
4	F	T	T	T	T
5	T	T	T	T	F

$\exists x \forall y \; \underline{((x \neq y) \rightarrow P(x,y))}$

$\forall x \exists y \; P(x,y)$

Everyone knows someone else's phone number.

x \ y	1	2	3	4	5
1	T	F	T	T	T
2	F	T	F	F	F
3	T	F	T	F	F
4	F	T	T	T	T
5	T	T	T	T	F

$\forall x \exists y \; ((x \neq y) \wedge P(x,y))$

Domain: Set of students in a class

$P(x,y)$: x knows y's phone number.

$K(x)$: x has the answer key.

Someone who has the answer key knows everyone else's phone number.

There is a person who has the answer key and knows everyone else's phone number.

$$\exists x \left(K(x) \wedge \forall y \left((x \neq y) \rightarrow P(x,y) \right) \right)$$

$$\exists x \, \forall y \left(K(x) \wedge \left((x \neq y) \rightarrow P(x,y) \right) \right)$$

Everyone who has the answer key knows someone's phone number.

$$\Rightarrow \forall x \left(K(x) \rightarrow \left(\exists y \; P(x,y) \right) \right)$$

$$\forall x \, \exists y \left(K(x) \rightarrow P(x,y) \right)$$

Everyone who has the answer key knows someone else's phone number.

$$\forall x \left(\underline{K(x)} \rightarrow \exists y \left((x \neq y) \wedge P(x,y) \right) \right)$$

$\exists x\, K(x)$

Expressing Uniqueness:

Exactly one person has the answer key.

Someone has the answer key and everyone else does not have the answer key.

$$\exists x\, (K(x) \wedge \forall y\, ((x \neq y) \rightarrow \neg K(y)))$$

$$\cancel{\forall x\, \neg K(x) \neq \neg \forall x\, K(x)}$$

Domain: runners in a race.

$A(x)$: x is an adult.
$C(x)$: x is a child.
$B(x,y)$: x beat y.

An adult won the race.

There is an adult who beat everyone else.

There is someone who is an adult who beat everyone else.

$$\exists x\,(A(x) \wedge \forall y\,((x \neq y) \rightarrow B(x,y)))$$

There is a child who beat every adult in the race.

$$\exists x\,(C(x) \wedge \forall y\,(A(y) \rightarrow B(x,y)))$$

Every adult beat every child in the race.

$$\forall x\,\forall y\,((A(x) \wedge C(y)) \rightarrow B(x,y))$$

adult child

There is a child who beat an adult in the race.

$$\exists x \, \exists y \, (C(x) \wedge A(y) \wedge B(x,y))$$

child adult

Sam beat exactly one person in the race.

$$\exists x \; B(Sam, x) \equiv \exists y \; B(Sam, y).$$

$$\exists x \, (B(Sam,x) \wedge \forall y \, ((y \neq x) \rightarrow \neg B(Sam,y)))$$

Sam beat exactly two people in the race.

Cannot do: $\forall A(x) \, \exists y \, B(x,y)$

$\forall x \, \exists y \; B(\underline{C(x)}, \underline{A(x)})$.

$\forall x \, \forall y \; B(x, y \neq x)$