$$\exists x \exists y \left(C(x) \wedge A(y) \wedge B(x,y) \right)$$

Note Title 1/13/2015

Company 1

	$T(x)$	$B(x)$
Joe	T	F
Belinda	T	F
Lucy	F	F
Brad.	F	T

$\exists x \left(T(x) \wedge B(x) \right) \Leftarrow$

$\equiv$

There is an executive who got a large bonus. Not true for Company 1

$\not\equiv$

$\exists x \left(T(x) \rightarrow B(x) \right) \Leftarrow$

True for Company 1

For example:

$T(\text{Brad}) \rightarrow B(\text{Brad})$ F T

$T(\text{Lucy}) \rightarrow B(\text{Lucy})$ F T.

Company 2

	$T(x)$	$B(x)$
Nancy	T	T
Ingrid	T	T
Jose	F	F
Ralph	F	T

$\forall x \left(T(x) \rightarrow B(x) \right)$

$\equiv$

Every executive got a large bonus. True for Company 2.

$\not\equiv$

$\underbrace{\forall x \left(T(x) \wedge B(x) \right)}_{\text{Not true for Company 2.}} \Leftarrow$

Counterexamples

$T(\text{Jose}) \wedge B(\text{Jose}) = F$ (F, F)

$T(\text{Ralph}) \wedge B(\text{Ralph}) = F$ (F, T)

Note Title 1/12/2015

De Morgan's Law for quantified statements.

- $\neg \exists x\, T(x) \equiv \forall x\, \neg T(x)$.

There does not exist a student in the class who is a transfer student.

Every student in the class is not a transfer student.

Consistent w/ DM's law on a finite domain:

$$\neg\big(\underbrace{T(a_1) \vee T(a_2) \cdots \vee T(a_n)}_{\equiv\ \neg E x\, T(x)}\big) \equiv \underbrace{\neg T(a_1) \wedge \neg T(a_2) \wedge \cdots \wedge \neg T(a_n)}_{\equiv\ \forall x\, \neg T(x)}.$$

$$\neg \forall\, T(x) \equiv \exists x\, \neg T(x)$$

It's not true that all the students are transfer students.

There is at least one student who is not a transfer student.

What is $\neg \forall x\,(x^2 > x) \equiv \exists x\, \neg(x^2 > x)$

$\exists x\,(x^2 \leq x)$

$\neg \exists x\,(x^2 = 2) \equiv \forall x\, \neg(x^2 = 2)$

$\forall x\,(x^2 \neq 2)$

Show: $\neg \forall x\,(P(x) \to Q(x)) \equiv \exists x\,(P(x) \wedge \neg Q(x))$

True for any domain & truth values.

[Can't do this w/ truth tables.]

$\neg \forall x\,(P(x) \to Q(x))$

$\exists x\, \neg(P(x) \to Q(x))$ DM.

$\exists x\, \neg(\neg P(x) \vee Q(x))$ Cond. Id.

$\exists x\,(\neg\neg P(x) \wedge \neg Q(x))$ DM.

$\exists x\,(P(x) \wedge \neg Q(x))$

De Morgan's Law with nested quantifiers:

When the negation moves from the left to the right of a quantifier, the quantifier changes: $\forall \to \exists$
$\exists \to \forall$.

Find a logically equivalent quantified statement in which negation signs immediately precede a predicate:

$$\neg \forall x \exists y \forall z \; P(x,y,z).$$

$$\equiv \exists x \neg \exists y \forall z \; P(x,y,z)$$

$$\equiv \exists x \forall y \exists z \; \neg P(x,y,z).$$

$$\neg \forall x \exists y \, (D(x) \to P(x,y)).$$

$$\exists x \forall y \; \neg (D(x) \to P(x,y))$$
$$\exists x \forall y \; \neg (\neg D(x) \vee P(x,y))$$
$$\exists x \forall y \; (\cancel{\neg\neg} D(x) \wedge \neg P(x,y))$$

$$\neg \exists x \forall y \, (T(x) \wedge S(x,y))$$

$$\forall x \exists y \; \neg (T(x) \wedge S(x,y))$$
$$\forall x \exists y \, (\neg T(x) \vee \neg S(x,y))$$

Note Title 1/20/2015

Argument:

$$\left.\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right\} \text{hypotheses}$$
$$\overline{\therefore\ c}$$

Conclusion.

$\therefore$ means "therefore".

An argument is <u>valid</u> if the conclusion is true whenever all the hypotheses are true.

Argument is valid if

$\underbrace{(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to c}$ is a tautology

For example:

$$\begin{array}{c} p \\ p \to q \\ \hline \therefore\ q \end{array}$$

p	q	$p \to q$	$p \wedge (p \to q)$	$\overbrace{(p \wedge (p \to q)) \to q}^{T}$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Here's an example of an argument that is <u>not</u> valid

$$\begin{array}{ll} \to\ p \to q & T \\ \to\ \neg p & T \\ \to\ \therefore\ \neg q & F \end{array}$$

$((p \to q) \wedge \neg p) \to \neg q$ is not a tautology.

$(p = F,\ q = T)$

Arguments are often expressed in English as in:

⇒ If I will study for my exam, then I will pass my exam.
<u>I will study for my exam.</u>
∴ I will pass my exam.

The <u>form</u> of an argument expressed in English is obtained by replacing each distinct proposition with a propositional variable:

- p : I will study for my exam.
- q : I will pass my exam.

$$\begin{array}{l} p \to q \\ \underline{p \qquad\qquad} \\ \therefore q \end{array}$$

An argument can be invalid even if the hypotheses & the conclusion are true:

If 5 is even, then 7 is even.
<u>5 is not even</u>
∴ 7 is not even.

$$\boxed{\begin{array}{l} p \to q \\ \underline{\neg p \qquad} \\ \therefore \neg q. \end{array}}$$

p : 5 is even.
q : 7 is even

In order for an argument to be valid, it must be true regardless of the actual truth values of the propositional variables.

p: 4 is a prime #.
q: 5 is a prime #.

This same argument could be used to conclude something that is not true:

$p \to q$
$\neg p$
$\therefore \neg q.$

T If 4 is a prime number then 5 is a prime number.
T 4 is not a prime number
F $\therefore$ 5 is not a prime number.

Another example:

$p \vee q$
$\neg p$
$\therefore q.$

Valid.

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

If both hyp are true, conclusion is true.

An alternative way to show an argument is valid: apply arguments that have already been shown to be valid.

(Much like showing logical equivalences using laws of propositional logic which are themselves logical equivalences already established via truth tables.)

$p \vee$

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array} \qquad \begin{array}{l} \neg r \\ w \to r \\ \hline \therefore \neg w \end{array}$$

There is a set of rules of inference that are useful for putting together valid arguments. (Go through them).

$$\begin{array}{l} p \to q \\ q \to r \\ \neg r \\ \hline \therefore \neg p. \end{array}$$

1.	$p \to q$	Hyp.
2.	$q \to r$	Hyp.
3.	$p \to r$	Hyp. Syll, 1, 2.
4.	$\neg r$	Hyp
5.	$\neg p$	Mod. Toll 3, 4.

Sometimes in reasoning it is clearer to translate English sentences into the language of logic and use rules of inference to reason.

If I have a hard workout, I am be sore. If I am sore I take aspirin. I am not taking aspirin.

w: workout
s: sore.
a: aspirin

$$\begin{array}{l} w \to s \\ s \to a \\ \neg a \\ \hline \therefore \neg w. \end{array}$$

If its not foggy or it doesn't rain then the race will be held and we will go see the race.

If the race is held, there will be a trophy ceremony.

The trophy ceremony was not held.

$\therefore$ It rained

f r c g t h

- also using identities from before.

$$(\neg f \vee \neg r) \to (c \wedge g)$$
$$c \to t$$
$$\neg t$$
$$\therefore r$$

$\neg(c \wedge g)$

$\neg c \vee \neg g$

1.	$c \to t$	Hyp.
2.	$\neg t$	Hyp.
3.	$\neg c$	Mod Toll 1 + 2.
4.	$\neg c \vee \neg g$	Addition 3.
5.	$\neg(c \wedge g)$	DM 4.
6.	$(\neg f \vee \neg r) \to (c \wedge g)$	Hyp.
7.	$\neg(\neg f \vee \neg r)$	Mod Toll 5, 6.
8.	$(\neg\neg f \wedge \neg\neg r)$	DM 7.
9.	$(\neg\neg f \wedge r)$	Doub Neg 8.
10.	r	Simpl 9.

$$\neg q$$
$$p \to q$$
$$\therefore \neg p.$$

$$\neg(c \wedge g)$$
$$(\neg f \vee \neg r) \to (c \wedge g)$$
$$\therefore \neg(\neg f \vee \neg r)$$