

Sections 3.1 & 3.2 (Language of Sets).

Note Title

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Set : an unordered collection of objects.

Objects in a set are called the elements of a set.

A set contains its elements.

$a \in A$ denotes a is an element of set A .
 A contains a .

If a is not an element of A , then $a \notin A$.

Specifying a set: Roster Method.

$$S = \{a, b, c, d\}.$$

The curly braces $\{ \}$ denote that order is not important:

$$\{ \underline{a}, \underline{b}, \underline{c}, \underline{d} \} = \{ \underline{c}, \underline{d}, \underline{a}, \underline{b} \} = S.$$

For large sets, ellipses (...) may be used when the pattern for filling in the missing members is clear:

$$\{1, 2, 3, \dots, 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\{a, b, c, d, \dots, z\}$$

Use the roster method to specify the following sets:

- The set of all vowels in the English alphabet.

$$\{a, e, i, o, u\}$$

- The set of all odd positive integers less than 10:

$$\{1, 3, 5, 7, 9\}$$

- The set of all odd positive integers less than 100:

$$\{1, 3, 5, \dots, 99\}$$

- The set of all integers less than 0:

$$\{\dots, -3, -2, -1\} = \{-1, -2, -3, \dots\}$$

Some important sets:

$$\mathbb{N} = \text{natural numbers} = \{1, 2, 3, \dots\}$$

$$\mathbb{Z} = \text{integers} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

$$\mathbb{Z}^+ = \text{positive integers} = \{1, 2, 3, \dots\}$$

$$\mathbb{R} = \text{real numbers}$$

$$\mathbb{R}^+ = \text{positive real numbers.}$$

$$\mathbb{C} = \text{complex numbers.}$$

$$\mathbb{Q} = \text{rational numbers.}$$

$$\text{positive integers } \{1, 2, 3, \dots\}$$

$$\text{non-negative integers } \{0, 1, 2, 3, \dots\}$$

Specifying a Set: Set-builder notation.

- Specify a domain and all the properties that the elements in the set must satisfy.

$$S = \{x \mid x \text{ is a positive integer less than } 100\}.$$

$$= \{x \in \mathbb{Z} \mid 0 < x < 100\}$$

$$= \{x \in \mathbb{Z}^+ \mid x < 100\}.$$

↑ read "such that".

More examples:

$$P = \{x \in \mathbb{Z} \mid x \text{ is prime}\}$$

$$\mathbb{Q} = \{x \in \mathbb{R} \mid x = a/b \text{ for some integers } a + b \text{ such that } b \neq 0\}$$

- Positive integer multiples of 3:

$$\{x \mid x = 3k \text{ and } k \in \mathbb{Z}^+\}$$

- Non-negative even integers:

$$\{x \mid x = 2k \text{ and } k \in \mathbb{Z} \text{ and } k \geq 0\}.$$



The empty set ($\emptyset = \{\}$) has no elements.

The universal set U contains everything currently under consideration.

- Sometimes implicit.
- Sometimes explicitly defined.

Subsets: A is a subset of B if and only if every element of A is also an element of B . $\forall x (x \in A \rightarrow x \in B)$

$$A \subseteq B.$$

For every a , $a \notin \emptyset$ therefore $\emptyset \subseteq S$
for every set S .

$$\forall x (x \in \emptyset \rightarrow x \in S)$$

$a \in S \rightarrow a \in S$ is always true; so $S \subseteq S$
for every set S .

To show $A \subseteq B$, need to show that $x \in A$ implies $x \in B$.

To show $A \not\subseteq B$, need to give a particular element x
such that $x \in A$ and $x \notin B$.

x is a counter-example to the claim that $A \subseteq B$.

Set Equality:

Two sets A & B are equal ($A = B$)

if and only if $\forall x (x \in A \leftrightarrow x \in B)$.

$$A = \{2, 4, 6, 8\}$$

$$A = C$$

$$B = \{1, 3, 5, 7, 9\}$$

$$B = D$$

$$C = \{x \in \mathbb{Z} \mid x \text{ is even and } 0 < x < 10\}$$

$$D = \{x \mid x = 2k+1 \text{ where } k \text{ is a non-negative integer less than } 5\}$$

$$\underline{A = B} \text{ iff } \forall x (x \in A \leftrightarrow x \in B) \text{ iff}$$

$$\forall x ((x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A))$$

$$\text{iff } \forall x (x \in A \rightarrow x \in B) \wedge \forall x (x \in B \rightarrow x \in A)$$

$$\text{iff } A \subseteq B \text{ and } B \subseteq A.$$

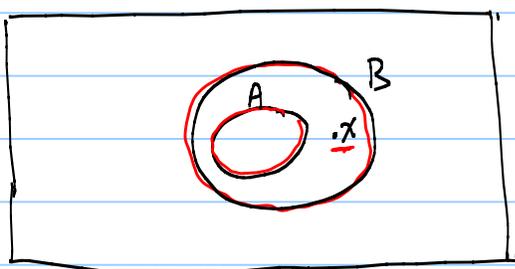
Definition: A is a proper subset of B, denoted $A \subset B$,

iff $A \subseteq B$ and $A \neq B$.

If $A \subset B$ then $\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \in B \wedge x \notin A)$

$A \subseteq B$ $x \leq y$.

$A \subset B$ $x < y$



$$A = \{2, 4, 6, 8\}$$

$$B = \{x \in \mathbb{Z} \mid x \text{ is even and } 0 < x < 10\}$$

$$C = \{x \in \mathbb{Z} \mid x \text{ is even and } 0 < x \leq 10\}$$

Which statements are true?

• $A \subseteq B$ yes.

• $A \not\subset B$

• $A \subseteq C$ yes.

• $A \subset C$

• $C \not\subseteq B$

• $A = C$

$$10 \in C$$

$$10 \notin B$$

$$10 \in C$$

$$10 \notin A$$

$10 \in C$ and $10 \notin A$.

For any two sets: If $A \subset B$ then $A \subseteq B$.

$A \subseteq B$ does not necessarily imply that $A \subset B$.

Set Cardinality:

Definition: If there are exactly n distinct elements in S , where n is a non-negative integer then S is finite.

Otherwise, S is infinite.

Definition: The cardinality of A , denoted $|A|$, is the number of (distinct) elements in A .

$$|\emptyset| = 0$$

S = the set of letters in the English alphabet:

$$|S| = 26$$

$$A = \{x \in \mathbb{Z} \mid x \text{ is even and } 0 < x \leq 100\}$$

$$|A| = 50$$

$$|\{1, 2, 3\}| = 3$$

$$S = \{1, 2, 3\}$$

$$|S| = 3$$

Can also have sets of sets:

$$S = \{ \phi, \{1\}, \{1,2\} \}$$

$$1 \notin S$$

$$\Rightarrow \{1\} \in S$$
$$\{1\} \notin S$$

$$|S| = 3$$

$$\{ \phi, \{1\} \} \subseteq S$$

$$\{ \{1\} \} \subseteq S$$

$$\{ \{1\} \} \subset S$$

Definition

Let A be a finite set. The power set of A , denoted $P(A)$ is the set of all subsets of A .

$$A = \{a, b\}$$

$$P(A) = \{ \phi, \{a\}, \{b\}, \{a, b\} \}$$
$$|P(A)| = 4$$

$$\text{If } |A| = n, \text{ then } |P(A)| = 2^n.$$

$$X \in P(A) \text{ iff } X \subseteq A.$$

$$A = \{ \Delta, 0, \square \}$$

$$\begin{aligned} & \{ \emptyset, \\ & \{ \Delta \}, \{ 0 \}, \{ \square \}, \\ & \{ \Delta, 0 \}, \{ 0, \square \}, \{ \Delta, \square \}, \\ & \{ \Delta, 0, \square \} \end{aligned}$$

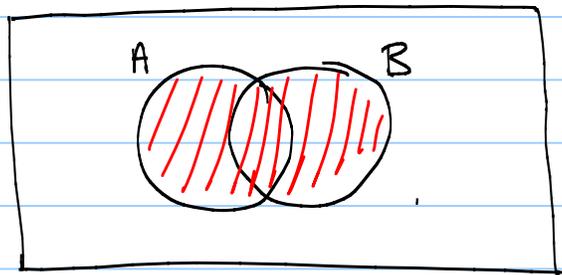
Set Operations.

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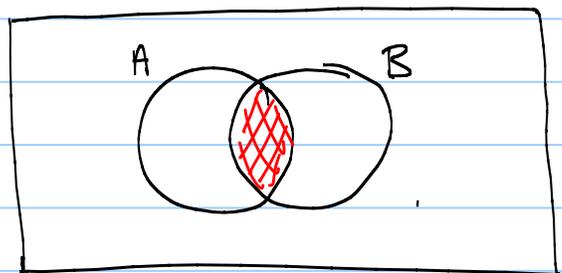
Definition: Let A & B be sets. The Union of A and B denoted $A \cup B$ is the set:
 $\{x \mid x \in A \vee x \in B\}$.

Ex: $A = \{1, 2, 3\}$. $A \cup B = \{1, 2, 3, 4, 5\}$
 $B = \{3, 4, 5\}$



Definition: Let A & B be sets. The Intersection of A and B denoted $A \cap B$ is the set:
 $\{x \mid x \in A \wedge x \in B\}$.

Ex: $A = \{1, 2, 3\}$. $A \cap B = \{3\}$
 $B = \{3, 4, 5\}$

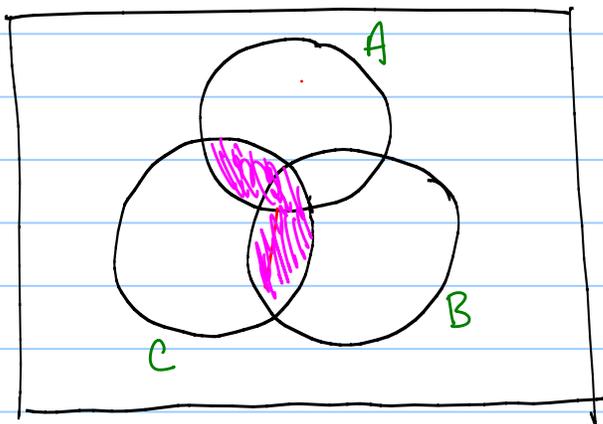


Can combine multiple set operations to define a set:

$$A \cup B \cap C.$$

Use parentheses to denote the order in which they are applied:

$$(A \cup B) \cap C$$



$$A \cup (B \cap C)$$

