

Homework 4

*Instructor: Sandy Irani***Covers Sections 3.1-3.6, 4.1-4.3.**

Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.

1. Challenge activities: 3.4.1, 3.4.2, 3.4.3, 4.2.1, 4.2.2, 4.3.1

2. Define the following sets:

- $A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}.$
- $B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}.$
- $C = \{4, 5, 9, 10\}$
- $D = \{3, 6, 9\}$
- $E = \{2, 4, 11, 14\}.$

Indicate which of the following statements are true:

- (a) $E \subseteq C$
- (b) $27 \in A$
- (c) $27 \in B$
- (d) $D \in A$
- (e) $D \subset A$
- (f) $A \subset A$
- (g) $A \subseteq A$
- (h) $15 \subset A$
- (i) $\{15\} \subset A$

3. What is the power set of $\{1\}$?

4. Give an example showing that set subtraction is not associative:

$$A - (B - C) \neq (A - B) - C.$$

5. List the elements of $\{0\} \times \{0, 1\}^2$.

6. Define the following sets as:

- $A = \{\text{tall, grande, venti}\}$
- $B = \{\text{foam, no-foam}\}$
- $C = \{\text{non-fat, whole}\}$

(a) Write any element from the set $A \times B \times C$.

(b) Write any element from the set $B \times C \times A$.

7. For each statement, indicate whether it is true or false. You can assume A , B and C are sets.

- (a) $A^2 \subseteq A^3$.
- (b) $A \subseteq A \times B$.
- (c) $A \cap A^2 = \emptyset$
- (d) If $A \subseteq B$, then $A \times C \subseteq B \times C$.

8. Find the following values:

- (a) $\lceil -4.999 \rceil$
- (b) $\lfloor -4.999 \rfloor$
- (c) $\lceil \frac{1}{2} + \lceil \frac{1}{2} \rceil \rceil$
- (d) $\lfloor -7 \rfloor$
- (e) $\lfloor .0001 \rfloor$
- (f) $\lfloor -.0001 \rfloor$

9. Which of the following are functions from \mathbb{R} to \mathbb{R} ? If f is a function, give its range.

- (a) $f(x) = \sqrt{x}$
- (b) $1/(x^2 - 4)$
- (c) $f(x) = \sqrt{x^2}$

10. Consider each of the following functions from \mathbb{Z} to \mathbb{Z} . Indicate whether the function is one-to-one, onto, neither or both.

- (a) $f(x) = \lceil \frac{x}{7} \rceil - 5$
- (b) $f(x) = 7x - 5$.

11. For each of the functions below, indicate whether the function is one-to-one, onto, both or neither. If the function is not onto, use set notation to specify its range. If the function is not one-to-one, give an example showing why.

- (a) $f : \mathbb{R} \rightarrow \mathbb{R}$. $f(x) = \lceil 5x \rceil$.
- (b) $f : \mathbb{R} \rightarrow \mathbb{R}$. $f(x) = x^2 + 1$.
- (c) $f : \mathbb{R} \rightarrow \mathbb{R}$. $f(x) = 3x - 5$.
- (d) A is defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. $f : P(A) \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. For $X \subseteq A$, $f(X) = |X|$.
- (e) A is defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. $f : P(A) \rightarrow P(A)$. For $X \subseteq A$, $f(X) = \overline{X}$.

12. For each of the functions below, indicate whether the function is one-to-one, onto, both or neither. If the function is not onto, use set notation to specify its range. If the function is not one-to-one, give an example showing why.
- (a) $f : \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by swapping the first and the last bits. For example $f(110) = 011$.
 - (b) $g : \{0, 1\}^3 \rightarrow \{0, 1\}^4$. The output of g is obtained by adding an extra copy of the first bit to the end of the string. For example $g(011) = 0110$.
 - (c) $h : \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of h is obtained by replacing the last bit with a copy of the first bit. For example $h(011) = 010$.