1. The diagram below shows a directed graph.

(a) What is the in-degree of \(d\)?
(b) What is the out-degree of \(c\)?
(c) What is the head of edge \((b, c)\)?
(d) What is the tail of edge \((g, f)\)?
(e) Is \(\langle a, g, f, c, d \rangle\) a walk in the graph? Is it a path?
(f) Is \(\langle a, g, f, d, b \rangle\) a walk in the graph? Is it a path?
(g) Is \(\langle c, g, f, e \rangle\) a circuit in the graph? Is it a cycle?
(h) Is \(\langle d, b, c, g, c, f, e, c, d \rangle\) a circuit in the graph? Is it a cycle?
(i) What is the longest cycle in the graph?
(j) Give an example of a cycle of length 4.
(k) Give an example of a path of length 5.
(l) Is there a path of length 3 from \(d\) to \(f\)? (If so, give the path)
(m) Is there a path of length 3 from \(a\) to \(c\)? (If so, give the path)
(n) Is it true that for each pair of vertices there is a walk from one to the other?
2. Here are two relations on the set \(a, b, c, d\):

- \(S = \{(b, b), (a, c), (c, d), (c, a)\}\)
- \(R = \{(b, c), (c, b), (a, d), (d, b), (c, a)\}\)

Give the relation \(R \circ S\). Express the relation as a set of pairs.

3. Define the following relations on the set \(\mathbb{R}\):

- \(R_1 = \{(x, y) : x \leq y\}\)
- \(R_2 = \{(x, y) : x > y\}\)
- \(R_3 = \{(x, y) : x < y\}\)
- \(R_4 = \{(x, y) : x = y\}\)

Use mathematical notation to describe the following relations:

(a) \(R_1 \circ R_2\)
(b) \(R_1 \circ R_3\)
(c) \(R_4 \circ R_2\)
(d) \(R_3 \circ R_4\)

4. The questions below refer to the graph \(G\) below:

(a) Is \((a, d)\) in \(G^2\)?
(b) Is \((a, d)\) in \(G^3\)?
(c) Is \((f, f)\) in \(G^3\)?
(d) Is \((b, b)\) in \(G^4\)?
(e) Is \((g, g)\) in \(G^3\)?
(f) Is \((e, f)\) in \(G^5\)?

5. Let \(G\) be a directed graph with 10 vertices. Let \(A\) be the adjacency matrix for \(G^3\) and \(B\) the adjacency matrix for \(G^4\). Describe in words what the product \(A \cdot B\) represents.
6. Give the adjacency matrix for the graph $G$ below.

![Graph G]

The use matrix multiplication to give the adjacency matrices for $G^2$, $G^3$, $G^4$, and $G^+$. 

7. Consider a digraph $G$ in which each vertex has in-degree at least one. Suppose that the relation defined by the edges of $G$ is symmetric. Is $G^+$ reflexive? Why or why not?

8. Draw a picture of $G^+$ for each of the digraphs below:

(a)

![Graph (a)]

(b)

![Graph (b)]