**ICS 6B** **Due: Friday, November 13, 2015, 5:30PM**

# Homework 6

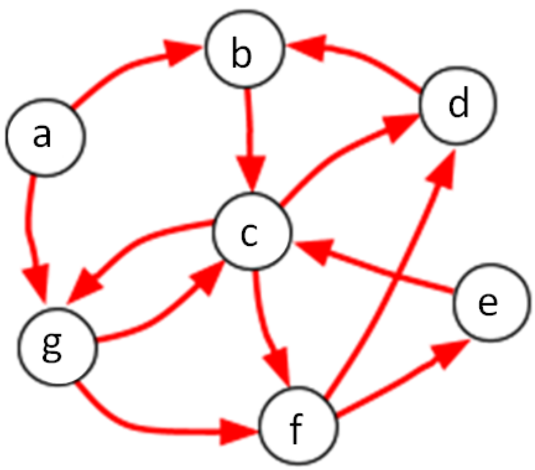
*Instructor: Sandy Irani*

**Covers Sections 6.3-6.6**

**There are no challenge activities for this material.**

**Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.**

1. The diagram below shows a directed graph.




   (a) What is the in-degree of $d$?

   (b) What is the out-degree of $c$?

   (c) What is the head of edge $(b, c)$?

   (d) What is the tail of edge $(g, f)$?

   (e) Is $\langle a, g, f, c, d\rangle$ a walk in the graph? Is it a path?

   (f) Is $\langle a, g, f, d, b\rangle$ a walk in the graph? Is it a path?

   (g) Is $\langle c, g, f, e\rangle$ a circuit in the graph? Is it a cycle?

   (h) Is $\langle d, b, c, g, c, f, e, c, d\rangle$ a circuit in the graph? Is it a cycle?

   (i) What is the longest cycle in the graph?

   (j) Give an example of a cycle of length 4.

   (k) Give an example of a path of length 5.

   (l) Is there a path of length 3 from $d$ to $f$? (If so, give the path)

   (m) Is there a path of length 3 from $a$ to $c$? (If so, give the path)

   (n) Is it true that for each pair of vertices there is a walk from one to the other?

2. Here are two relations on the set a, b, c, d:

   - $S = \{(b, b), (a, c), (c, d), (c, a)\}$
   - $R = \{(b, c), (c, b), (a, d), (d, b), (c, a)\}$

   Give the relation $R \circ S$. Express the relation as a set of pairs.

3. Define the following relations on the set $\mathbb{R}$:

   - $R_1 = \{(x, y) : x \leq y\}$
   - $R_2 = \{(x, y) : x > y\}$
   - $R_3 = \{(x, y) : x < y\}$
   - $R_4 = \{(x, y) : x = y\}$

   Use mathematical notation to describe the following relations:

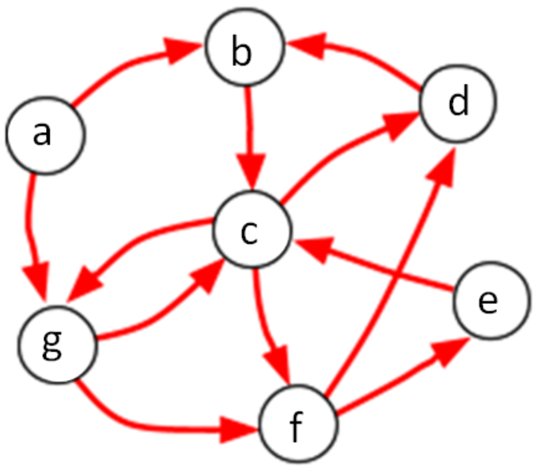
   (a) $R_1 \circ R_2$

   (b) $R_1 \circ R_3$

   (c) $R_4 \circ R_2$

   (d) $R_3 \circ R_4$

4. The questions below refer to the graph $G$ below:




   (a) Is $(a, d)$ in $G^2$?

   (b) Is $(a, d)$ in $G^3$?

   (c) Is $(f, f)$ in $G^3$?

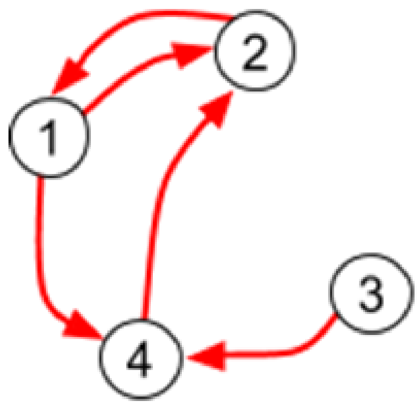
   (d) Is $(b, b)$ in $G^4$?

   (e) Is $(g, g)$ in $G^3$?

   (f) Is $(e, f)$ in $G^5$?

5. Let $G$ be a directed graph with 10 vertices. Let $A$ be the adjacency matrix for $G^3$ and $B$ the adjacency matrix for $G^4$. Describe in words what the product $A \cdot B$ represents.

6. Give the adjacency matrix for the graph $G$ below.

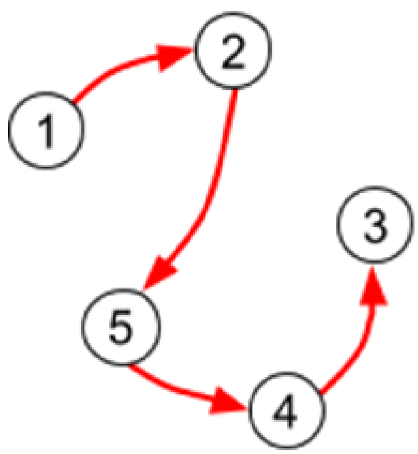



The use matrix multiplication to give the adjacency matrices for $G^2$, $G^3$, $G^4$, and $G^+$.

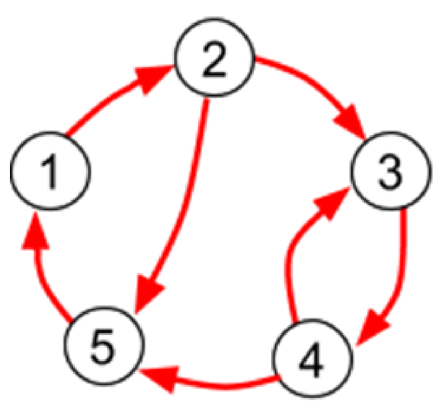
7. Consider a digraph $G$ in which each vertex has in-degree at least one. Suppose that the relation defined by the edges of $G$ is symmetric. Is $G^+$ reflexive? Why or why not?

8. Draw a picture of G+ for each of the digraphs below:

(a)




(b)



(c)