1. *(10 points)* A student club holds an election for officers. Before the voting, members can nominate each other. It is also possible for a member to nominate himself or herself. Some of the members are new members. Some of the members are currently officers. The domain is the set of members of the club. One of the members of the club is named Sam. Define the following predicates:

   - $N(x, y)$: person x nominated person y for a position.
   - $W(x)$: person x is a new member.
   - $O(x)$: person x is currently an officer.

   Give a quantified expression that is logically equivalent to each of the folloiwng statements:

   (a) All the new members nominated all the officers.

   (b) One of the current officers did not nominate anyone.

   (c) Everyone nominated someone.

   (d) Someone nominated everyone.

   (e) Everyone nominated someone besides themselves. (This statement does not specify whether anyone nominated themselves or not).

   (f) (*Extra credit: 2 points)* Exactly one person nominated Sam.

2. *(6 points)* Circle the statements that are true. The domain for all the variables is the set of real numbers.

   (a) $\forall x \exists y \ \ x + y = 0$

   (b) $\exists x \forall y \ \ x + y = 0$

   (c) $\exists x \forall y \ \ xy = y$

   (d) $\exists x \exists y \ ((x^2 = y^2) \wedge (x \neq y))$

   (e) $\forall x \forall y \exists z \ \ z = (x + y)/2$

   (f) $\forall x \exists y \forall z \ \ z = (x + y)/2$

3. *(9 points)* Below is a partial proof of the argument whose hypotheses are $\exists x(\neg P(x) \wedge T(x))$ and $\forall x(P(x) \vee Q(x))$ and whose conclusion is $\exists x(Q(x) \wedge T(x))$. Fill in the proof that the argument is valid. You need to fill in the missing steps on the left and rules of inference on the right. For each rule you fill in, indicate the row number(s) it applies to.

1. $\exists x(\neg P(x) \wedge T(x))$	Hypothesis
2. $\neg P(c) \wedge T(c)$, for some c	
3.	Simplification, 2
4. $\forall x(P(x) \vee Q(x))$	Hypothesis
5.	Universal instantiation, 4
6. $Q(c)$	
7. $T(c)$	
8.	Addition, 6, 7
9. $\exists x(Q(x) \wedge T(x))$	

4. *(8 points)* For each argument below, indicate whether it is valid or invalid. If an argument below is valid, then it uses exactly one of the rules of inference. Indicate which rule is used for the argument. If the argument is invalid, give truth assignments to the propositions "Sally took the medication" and "Sally had side effects" that prove the argument is invalid. For example, an answer might be: Invalid. Sally took the medication = F, Sally had side effects = T.

(a)
Sally had a side effect or Sally took the medication.
Sally took the medication.
$\therefore$ Sally did not have side effects.

(b)
If Sally took the medication then she had side effects.
Sally did not take the medication.
$\therefore$ Sally did not have side effects.

(c)
Sally took the medication.
$\therefore$ Sally took the medication or had side effects.

(d)
If Sally had side effects, then she took the medication.
Sally did not take the medication.
$\therefore$ Sally did not have side effects.

5. *(9 points)* Below is an outline of the steps of the proof that $\sqrt{2}$ is irrational. However, the steps are in not in the correct order. Write down the letter corresponding to each step to show the correct order for the statements. The proof uses the fact proven in class that if n is an integer and n^2 is even, then n is also even.

   A. Since n is even, $n = 2k$ for some integer k. Plug in $2k$ for n: $n^2 = (2k)^2 = 2d^2$.

   B. Since n and d are both even, 2 evenly divides n and d.

   C. Express $\sqrt{2}$ as n/d, where n and d are integers, $d \neq 0$ and there is no integer greater than 1 that evenly divides both n and d.

   D. Therefore $\sqrt{2}$ must be irrational.

   E. Square both sides of the equation $\sqrt{2} = n/d$ and multiply by 2 to get $n^2 = 2d^2$. Since n^2 is an integer multiple of 2, n^2 is even and therefore n is even.

   F. Assume that $\sqrt{2}$ is rational.

   G. Since d^2 is even, d is also even.

   H. This contradicts the assumption that there is no integer greater than 1 that divides both n and d.

   I. Since $4k^2 = 2d^2$, we can divide both sides by 2 and conclude that d^2 is also even.

   ________ ________ ________ ________ ________ ________ ________ ________ ________

(8 points) For the next four questions, your choices will be one of the following six statements. Any of the statements could potentially be the correct answer for more than one question.

A. x is rational and y is rational.

B. x is irrational and y is irrational.

C. $x + y$ is rational.

D. $x + y$ is irrational.

E. x is irrational or y is irrational.

F. x is rational or y is rational.

Theorem 1. *For any two real numbers, if $x + y$ is irrational then x is irrational or y is irrational.*

Statement 1: A direct proof of Theorem 1 would assume for real numbers x and y that __(+)__and would prove that __(*)__.

6. What expression should go in the space labeled (+) in Statement 1?

7. What expression should go in the space labeled (*) in Statement 1?

Statement 2: A proof by contrapositive of Theorem 1 would assume for real numbers x and y that __(&)__and would prove that __($)__.

8. What expression should go in the space labeled (&) in Statement 2?

9. What expression should go in the space labeled ($) in Statement 2?