

Version 1

## Test 3

ICS 6B

Fall 2015

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## Instructions:

1. Wait until instructed to turn over the cover page.
2. Write your name on your Scantron form.
3. Write your student ID number in the box labeled "Subject" on your scantron form.
4. Write the version of your test (1, 2, or 3) in the box labeled "TEST NO" on your scantron form.
5. There are three pages (not including this cover sheet) and 40 questions in this test.
6. You only need to turn in your scantron form at the end of the test. You are advised to keep your copy of the test so that you can check your answers since there will not be anything turned back to you showing which questions you got right.

For the next 7 questions, select from the following four choices to describe the given function.

A. Neither onto nor one-to-one.

B. One-to-one, but not onto.

C. Onto, but not one-to-one.

D. One-to-one and onto.

1.  $d : \{0, 1\}^4 \rightarrow \{0, 1\}^4$ .  $d(x)$  is obtained from  $x$  by removing the second bit and placing it the end. For example  $d(1011) = 1110$ .

2.  $f : \{0, 1\}^4 \rightarrow \{0, 1\}^4$ .  $f(x)$  is obtained from  $x$  by replacing the last bit with 1. For example  $f(1000) = 1001$ .

3.  $g : \{0, 1\}^4 \rightarrow \{0, 1\}^3$ .  $g(x)$  is obtained from  $x$  by removing the first bit. For example  $g(1000) = 000$ .

4.  $h : \mathbb{Z} \rightarrow \mathbb{Z}$ .  $h(x) = \lceil \frac{x}{3} \rceil$ .

5.  $h : \mathbb{Z} \rightarrow \mathbb{Z}$ .  $h(x) = x + 3$ .

6.  $h : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ .  $h(x) = x + 3$ .

7.  $h : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ .  $h(x, y) = |x| - |y|$ .

The next set of questions refer to the definitions of  $d$ ,  $f$  and  $g$  from questions 1-3.

8. What is  $d^{-1}(1001)$ ?

A. 1010

C. 1100

B. 1001

D. Not well defined.

9. Which of the following functions is not well defined?

A.  $d \circ f$

C.  $g \circ d$

B.  $f \circ g$

D.  $f \circ d$

10. What is the range of  $g \circ f$ ?

A.  $\{0, 1\}^4$

C.  $\{001, 011, 101, 111\}$

B.  $\{0, 1\}^3$

D.  $\{0, 1\}^3 \times \{1\}$

11. What is  $f \circ d(1011)$ ?

A. 1111

C. 1011

B. 1110

D. 0111

Select the correct value for the following expressions:

12.  $\lfloor -4.1 \rfloor =$

A. 4.1

B. -4

C. -4.1

D. -5

13.  $\lfloor \frac{5}{2} + \lceil \frac{5}{4} \rceil \rfloor =$

A. 2

B. 4

C.  $\frac{15}{4}$

D. 3

Questions 14 through 31, refer to the set definitions given below. For questions that use the complement operation, you can assume that the universe is the set of all integers.

•  $A = \{1, 2, 3, 4\}$

•  $B = \{x \in \mathbb{Z} : x \text{ is even}\}$

•  $C = \{-1, -2, -3\}$

•  $D = \{x \in \mathbb{Z}^+ : x \text{ is prime}\}$

•  $E = \{x \in \mathbb{Z}^+ : x < 5\}$

•  $F = \{x \in \mathbb{Z}^+ : x \leq 5\}$

•  $G = \{-1, 0, 1, 2, 3\}$

•  $H = \{2, 3, 4, 5, 6, 7\}$

For questions 14 through 28, indicate whether each statement is true or false. Select "A" for true and "B" for false.

14.  $C \subseteq C \times C$

19.  $E \subseteq F$

24.  $|(G - H) \cap A| = 1$

15.  $(-3, 2) \in A \times C$

20.  $B \times D \subset \mathbb{Z} \times \mathbb{Z}$

25.  $H \subseteq B \cup D$

16.  $\emptyset \in P(C)$

21.  $\{-1, -3\} \subseteq P(C)$

26.  $1 \in A \cap G \cap H$

17.  $\emptyset \in A$

22.  $B \cap F = B \cap E$

27.  $G \cup H \subseteq \mathbb{R}$

18.  $A \subset E$

23.  $G \cap H \subset A$

28.  $\overline{B} \cap G = \{1, 3\}$

29. Which of the following choices is a subset of  $P(A)$ ?

A. 1

B.  $\{1\}$

C.  $\{\{1\}\}$

30. What is  $(C \cap G) \cup A$ ?

A.  $\{-1, 0, 1, 2, 3, 4\}$

B.  $\{-1, 1, 2, 3, 4\}$

C.  $\{1, 2, 3\}$

D.  $\{-1\}$

31. What is  $\overline{(G \cap H)} \cap A$ ?

A.  $\emptyset$

B.  $\{1, 4\}$

C.  $\{2, 3\}$

D.  $\{1, 2, 3, 4\}$

Define the relation  $P$  on  $\mathbb{R}$  such that for  $x, y \in \mathbb{R}$ ,  $x$  is related to  $y$  if and only if  $\lceil x \rceil \leq \lceil y \rceil$ .

32. Which of the following statements is true?

- A. The relation  $P$  is reflexive.
- B. The relation  $P$  is anti-reflexive.
- C. The relation  $P$  is neither reflexive nor anti-reflexive.

33. Which of the following statements is true?

- A. The relation  $P$  is symmetric.
- B. The relation  $P$  is anti-symmetric.
- C. The relation  $P$  is neither symmetric nor anti-symmetric.

34. Which of the following statements is true?

- A. The relation  $P$  is transitive.
- B. The relation  $P$  is not transitive.

Now define two relations below whose domain is the set of all UCI students.

- S: Student  $x$  is related to student  $y$  if  $x$  has fewer letters in his or her last name than  $y$ .
- T: Student  $x$  is related to student  $y$  if  $x$  and  $y$  have the same first **or** the same last name (or both).

You can assume that any combination is possible: there are students who have the same first name but different last names, students who have the same last name but different first names, students who have the same first and last names and students who have no names in common.

35. Which of the following statements best describes relation  $S$ ?

- A. The relation  $S$  is reflexive.
- B. The relation  $S$  is anti-reflexive.
- C. The relation  $S$  is neither reflexive nor anti-reflexive.

36. Which of the following statements best describes relation  $S$ ?

- A. The relation  $S$  is symmetric.
- B. The relation  $S$  is anti-symmetric.
- C. The relation  $S$  is neither symmetric nor anti-symmetric.

37. Which of the following statements best describes relation  $S$ ?

- A. The relation  $S$  is transitive.
- B. The relation  $S$  is not transitive.

38. Which of the following statements best describes relation  $T$ ?

- A. The relation  $T$  is reflexive.
- B. The relation  $T$  is anti-reflexive.
- C. The relation  $T$  is neither reflexive nor anti-reflexive.

39. Which of the following statements best describes relation  $T$ ?

- A. The relation  $T$  is symmetric.
- B. The relation  $T$  is anti-symmetric.
- C. The relation  $T$  is neither symmetric nor anti-symmetric.

40. Which of the following statements best describes relation  $T$ ?

- A. The relation  $T$  is transitive.
- B. The relation  $T$  is not transitive.