1. Prove that the eigenvalues of a unitary operator can be written in the form $e^{i\theta}$ for some real $\theta$.

2. Prove that two eigenvectors of a Hermitian matrix with different eigenvalues are orthogonal.

3. Prove that a normal matrix is Hermitian if and only if it has real eigenvalues.

4. The Hadamard operator on one qubit may be written as 
   
   $$H = \frac{1}{\sqrt{2}} [(|0\rangle + |1\rangle)(0\rangle + (|0\rangle - |1\rangle)(1\rangle)].$$

   Give a closed form expression for $H^\otimes n$ using outer-bracket notation in the standard basis.

5. Consider the quantum state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + e^{i\theta}|1\rangle)$. Describe a measurement that will yield some information about the phase $\theta$ so that if you are given many copies of $|\psi\rangle$ you can determine $\theta$ to arbitrary accuracy.

6. Suppose that $A'$ and $A''$ are matrix representations of linear operator $A$ on a vector space $V$ with respect to two different orthonormal bases $|v_i\rangle$ and $|w_i\rangle$. The elements of $A'$ and $A''$ are $A'_{ij} = \langle v_i | A | v_j \rangle$ and $A''_{ij} = \langle w_i | A | w_j \rangle$. Characterize the relationship between $A'$ and $A''$. 