Quantum Teleportation

The No-cloning theorem tells us that there is no unitary transform that does $|4\rangle \otimes |0\rangle \rightarrow |4\rangle \otimes |4\rangle$ for all $|4\rangle$.

However, if we are willing to destroy the original, we can transmit a qubit using an entangled pair (plus 2 bits of classical communication).

Alice would like to transmit $|4\rangle$ (single qubit state) to Bob. Alice and Bob share the entangled state $|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$.

Alice has the first qubit and Bob has the second.

$$|4\rangle = a_0 |0\rangle + a_1 |1\rangle$$

Alice

Bob

Start with $|4\rangle \otimes |\Phi^+\rangle = (a_0 |0\rangle + a_1 |1\rangle) \otimes \left( \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \right) = \sum_{i=0,1} \sum_{j=0,1} \frac{1}{2} a_i a_j |i\rangle |j\rangle |i\rangle |j\rangle$

After CNOT

$$\sum_{i=0,1} \sum_{j=0,1} \frac{a_i a_j}{\sqrt{2}} |i, i\rangle |j, j\rangle$$

Now Alice measures hidden qubit.

$$l = 0 \quad (i = j) \quad \sum_{j=0,1} \frac{a_j}{\sqrt{2}} |j, j\rangle \rightarrow a_0 |00\rangle + a_1 |11\rangle$$

normalize
\[ l = 1 \quad (i \neq j) \quad \frac{a_0}{\sqrt{2}} |00\rangle + \frac{a_1}{\sqrt{2}} |10\rangle \quad \text{normalize} \quad a_0 |00\rangle + a_1 |10\rangle \]

Alice sends \( l \) to Bob.

If \( l = 1 \), Bob will toggle his qubit:

\[ a_0 |00\rangle + a_1 |11\rangle \]

Almost done, except that Alice's qubit is still entangled with Bob's. If she does a measurement, then it will destroy Bob's copy.

Alice sends measurement result to Bob

Bob does nothing

Bob does phase flip

\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

\[ |z\rangle = (a_0 |00\rangle + a_1 |11\rangle) \]