1. Show that the class \( \text{ZPP} = \text{RP} \cap \text{co-RP} \).

2. Describe a decidable language that is in \( \text{P/poly} \) but not in \( \text{P} \).

3. The language \( \text{USAT} \) is the set of boolean formulae that have a unique satisfying assignment. In class we proved the Valiant-Vazirani theorem which says that that there exists a polynomial-time algorithm \( f \) such that for every \( n \)-variable boolean formula, \( \phi \)

\[
\begin{align*}
\phi \in \text{SAT} & \Rightarrow P[f(\phi) \in \text{USAT}] \geq \frac{1}{8n} \\
\phi \notin \text{SAT} & \Rightarrow P[f(\phi) \in \text{SAT}] = 0.
\end{align*}
\]

Now suppose we have a polynomial time algorithm that given a boolean formula \( \phi \), will answer "yes" if \( \phi \in \text{USAT} \), will answer "no" if \( \phi \notin \text{SAT} \) and will answer arbitrarily otherwise. Prove that this would imply that \( \text{RP} = \text{NP} \).

4. A language \( L \subseteq \{0, 1\}^* \) is sparse if there is a polynomial \( p \) such that \( |L \cap \{0, 1\}^n| \leq p(n) \) for all \( n \). Show that every sparse language is in \( \text{P/poly} \).

5. Define \( \text{ZPP}' \) to be the class of all languages decided by a probabilistic Turing Machine running in expected polynomial time. That is, for every language \( L \) in \( \text{ZPP}' \) there is a probabilistic Turing Machine \( M \) (with two read-only tapes the first tape containing the input, and the second tape containing a random bit in every tape square) with the following behavior: on input \( x \in L \), \( M \) always accepts; on input \( x \notin L \), \( M \) always rejects; and for every input \( x \),

\[
E[\# \text{ steps before } M \text{ halts}] = |x|^{O(1)}.
\]

Show that \( \text{ZPP}' = \text{ZPP} \).

6. The class \( \text{P/log} \) is the class of languages decidable by a Turing Machines running in polynomial time that take \( O(\log n) \) bits of advice. Show that \( \text{SAT} \in \text{P/log} \) implies \( \text{P} = \text{NP} \).